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The pendulum of the hand of statistics and engineering

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Abstract. - In the following work, you will find the research carried out to understand in a more feasible way the analysis of the simple pendulum since this study helps to understand many aspects found in everyday life, such as the operation of a clock. For this, a model was made to carry out all the necessary measures to process the data obtained. It was observed that there is a variation of the average time depending on its length, so applying statistical principles, where taking several samples, it was possible to visualize a natural phenomenon, applying an analysis based on engineering for gravity calculation, analytically and graphically.

Keywords: Gravity, statistics, physics, measurement, pendulum.

El péndulo de la mano de la estadística y la ingeniería

Resumen: En el siguiente trabajo encontrarás las investigaciones realizadas para entender de una manera más factible el análisis del péndulo simple, ya que este estudio ayuda a entender muchos aspectos que se encuentran en la vida cotidiana, como el funcionamiento de un reloj, para lo cual se realizó un modelo que permitió realizar todas las mediciones necesarias para luego procesar los datos obtenidos. Se observó que existe una variación del tiempo promedio como consecuencia de su duración, para lo cual, aplicando principios estadísticos, donde tomando varias muestras, fue posible visualizar un fenómeno natural, aplicando un análisis basado en ingeniería, para el cálculo de la gravedad, de manera analítica y gráfica.

Palabras clave: Gravedad, estática, física, medición, péndulo.

I. INTRODUCTION

The pendulum is a severe body that can oscillate suspended from a point by a thread or rod. This has a mass, so, in turn, it generates a force that attracts it toward the gravitational center. There are several variations of the system. These can be formed from different materials. However, they are all governed by the same principle, oscillating and performing in certain circumstances, movements considered periodic or quasi-periodic [1]. In this sense, the pendulum represents the oscillatory movements of physics. The pendulum describes a circular trajectory. However, the arc it generates will have the radius of the length of the thread, being the pendulum at the most significant angle at which the weight will be thrown; this has potential energy, which will be converted into kinetic energy until reaching the equilibrium point (when it has an angle of 90° with the horizontal), When it begins to rise to the greater angle it becomes potential again.

One of the most valuable tools for analyzing the behavior of simple pendulum variables is descriptive statistics, which contributes significantly to most engineering work. Descriptive statistics is a branch used to summarize and present data clearly and concisely. In simple pendulum-based work, measurements of pendulum swing times can be summarized by descriptive statistics and presented as tables, graphs, and statistics as mean median, and standard deviation. This allows for a better understanding of the data and a more accessible interpretation of experimental results.

In this work, the behavior of gravity in the simple pendulum was analyzed with statistical applications. For this, an experimental practice of the pendulum was carried out to take the necessary data to evaluate the severity values later and make the respective calculations of errors and statistical analysis. Ten length measurements and three-time values have been considered for each case to optimize calculation procedures.

This work consists of 4 sections; in the first, the fundamentals of the subject of study have been described; in the second, the theoretical elements that support this research will be raised; in the third section, we will proceed to explain the methodological processes of the experiment. Finally, the results and conclusions are presented.

II. DEVELOPMENT

Ideally, a simple pendulum has a mass (m) suspended from a wire of length (l), inextensible and without mass [1]. The mass moves through an angle θ with the vertical axis. In our case, this angle should not exceed 15°, and the oscillation, without imparting an initial speed, is allowed to move only under the restrictions imposed by gravity and rope.

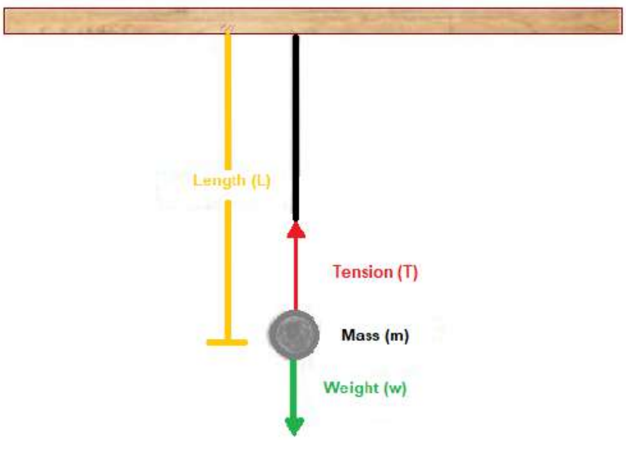

Fig. 1. Simple pendulum in balanced position.

Therefore, the particle's motion is confined within an arc of radius (θ) in the plane. The only forces acting on this mass are the weight ($\vec{w}$) and the string's tension ($\vec{T}$).

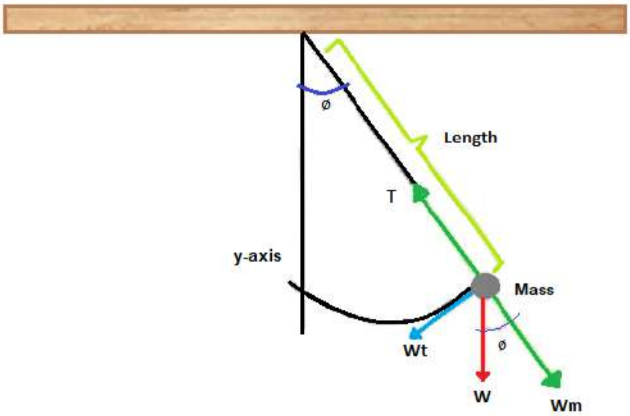

Fig. 2. Physical relationship of a simple module

Some movements are constant in physics and everyday life, such as tides, heartbeats, and clocks. Still, this last is the most thought for analyzing a simple pendulum because it is the most obvious example of periodic movement. Periodic motion is the movement of a body from one side to the other along a fixed path, returning to each position and velocity after a fixed time interval. A simple pendulum consists of an insignificant mass suspended from a string [2].

Any periodic motion can be thought of as the result of a set of simultaneous simple harmonic oscillatory movements. For this reason, simple harmonic motion is the basis for studying all periodic motion and, therefore, all periodic phenomena. The actual period and frequency are obtained in current practice, and the pendulum formula obtains the theoretical frequency.

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

where T = period (s), l = length and g = gravity m/s.

For laboratory analysis, single pendulum oscillators are idealized as natural systems with less than fifteen-degree angles. Due to their relative vibration relative to equilibrium, they generate mechanical energy dissipated in the form of kinetic energy. Based on equilibrium calculations, kinetic energy at its peak is idealized to identify its other measurements, such as amplitude, frequency, period, rapidity, and quality [3].

Gravity and oscillation are two fundamental parts of this study. However, it is known that gravity is the force that attracts objects to the earth's center. While oscillation is the movement of an object between two certain positions, these oscillations can vary depending on the length of the thread or material that holds it suspended. Due to this, it was decided that the best way to check everything described above would be to make a model that is functional, that allows and perform the analysis of the oscillation and the change that exists in it according to the length of the thread that helps to keep the dough suspended. This model aims to verify that what is in theory can be put into practice. On the other hand, you want to implement automation to it in a way that facilitates the collection of data and the necessary calculation with them.

A. Descriptive statistics in engineering

Statistics is essential to engineering, providing a solid foundation for decision-making and problem-solving. In experiment design, statistics is used to plan, design, and analyze experiments to determine relationships between variables and to optimize processes. For example, engineers use statistical techniques in chemical process engineering to optimize reaction conditions and maximize process efficiency.

In addition, in quality control, statistics are used to control and improve the quality of products and processes through statistical techniques such as process control and process capability. This allows engineers to detect and correct quality issues early, reducing costs and increasing customer satisfaction. In maintenance engineering, statistics are used to plan and schedule preventive and predictive maintenance of machinery and equipment, helping to reduce costs and increase asset availability. In short, statistics is a valuable tool for engineering as it allows engineers to collect, analyze, and use data to make informed decisions and solve problems in various fields.

The main errors to analyze in this paper are:

1. Absolute error: the difference exists between the measurements' theoretical and practical values obtained when making the measurements [1].

2. Systematic error: it varies predictably; this means you have an idea of the error that will come out [2].

3. Zero error: this error is one of the most common on a day-to-day basis; many times, it is due to a factory error; this happens when the equipment is zero; it marks a value that it should not.
4. Non-linearity error happens when the results do not generate a straight line but have a nonlinear trend [3].

5. Standard deviation: determines the variation between the data and the mean; when it is low, the data is concentrated near the mean. A high deviation indicates that the data is distributed over a broader range [4].

6. Variance: is an indicator of how uneven the data are around the mean; the higher the average, the greater the dispersion of the data and the less representative the mean [7].

III. METHODOLOGY

For the experimental data, the data collection was carried out with the prepared model and a body of 7 grams of weight at 15 degrees of inclination.

The model includes a design to build a model in a physical or a simple oscillatory pendulum. It has a wooden base to keep it stable, a wooden bar 1m 30cm high with a crossbar at the tip to hold a string with a weight attached as presented in the fig. 3.

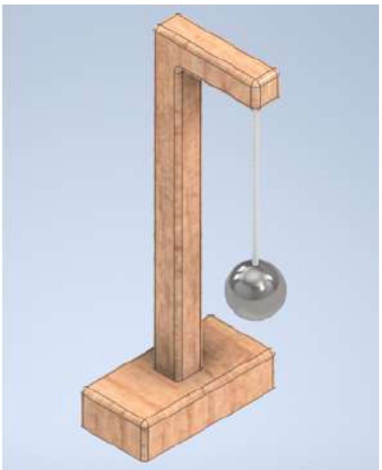

Fig. 2. Physical relationship of a simple module

After making the model as presented in the schematic diagram, a weight of 7 grams was incorporated with a diameter of 3 cm attached to a string of initial length of 10 cm, which is varied for different lengths, increasing from 5 cm to 5 cm after each measurement, in turn, to determine the time the pendulum was positioned at an angle of 15°, where the weight will be released to start with the taking of time until a complete oscillation ends, to later continue with the analysis of the average and obtain a more accurate measurement.

For the theoretical data, data were taken from a simulator at the University of Colorado, where the weight of the body was prepared as 0.10 kg; due to the minimum mass limitation of the system, the same inclination was adjusted and taken at a slow speed for greater precision of the time in which the cycle is completed.

Once the data was obtained, it was entered into an Excel table. With the experimental data collected at three different times, an average of the three values was obtained (Table 1)

Table 1. Physical relationship of a simple module

Theoretical		Experimental			
L (cm)	**T (s)**	**L (cm)**	**T_1(s)**	**T_2 (s)**	**T_3 (s)**
10	0.69	10	0.76	0.6	0.67
15	0.85	15	0.87	0.85	0.83
20	1	20	0.93	0.99	1
25	1.13	25	1.06	1.03	1.05
30	1.24	30	1.13	1.1	1.15
35	1.31	35	1.2	1.33	1.32
40	1.44	40	1.34	1.5	1.37
45	1.54	45	1.4	1.48	1.53
50	1.61	50	1.55	1.57	1.6
55	1.66	55	1.67	1.69	1.66
60	1.74	60	1.68	1.67	1.7
65	1.83	65	1.85	1.82	1.86
70	1.91	70	1.9	1.92	1.98

IV. RESULTS

Once the experiment was performed, the following results were found:

Regarding the calculation of errors, it can be seen in Table 2. That the absolute error is very little because the measures taken did not significantly differ from one to the other. Therefore, it is considered that the values are the most accurate possible.

For absolute error calculations, the formula is implemented:

$$EA = X_0 - X \quad (2)$$

The absolute error presented a range from 0.0133s to 0.0233s, representing the study. On the other hand, the zero error was 0.68s.

On the other hand, in Table 2, you can see the nonlinearity errors obtained when evaluating the values in the formulas found and the period and severity calculations; these values help to understand the system's behavior.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (3)$$

$$y = mx + b \qquad (4)$$

$$\frac{L_2 - L_1}{(T_2)^2 - (T_1)^2} = \frac{g}{4\pi^2} \qquad (5)$$

$$T = 2\pi \times \sqrt{\frac{L}{g}} \qquad (6)$$

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Table 2. Physical relationship of a simple module

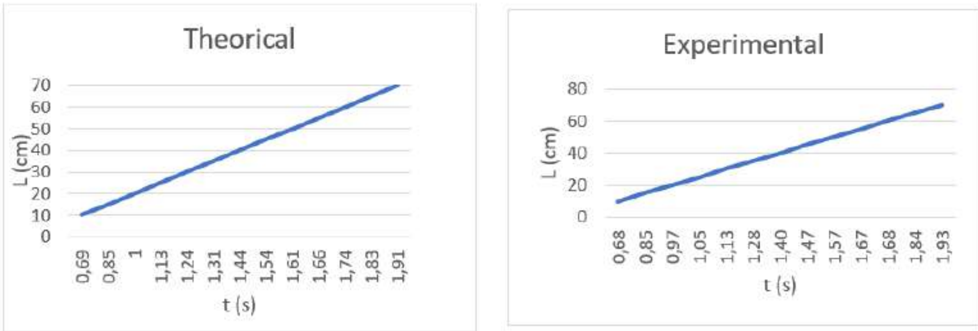
Non-linearity error				
Straight	Y (experimental)	Y (theoretical)	Difference	Difference between points
ye=48x - 22.64	9.94	10.2442	0.3042	0.304492065
	18.26	18.113	-0.147	0.147
	24.18	25.49	1.31	1.310271388
	27.7	31.8834	4.1834	4.184229918
	31.54	37.2932	5.7532	5.754316179
	39.06	40.7358	1.6758	1.676012157
	44.82	47.1292	2.3092	2.309491088
yt=49.18x - 23.69	48.02	52.0472	4.0272	4.027808317
	52.98	55.4898	2.5098	2.510067825
	57.78	57.9488	0.1688	0.169325774
	58.26	61.8832	3.6232	3.623643105
	65.94	66.3094	0.3694	0.369640552
	70.26	70.2438	-0.0162	0.028405711

For calculating the lines, equations (3) to (6) were used, allowing the use of both theoretical and experimental data.

For the calculation of gravity, equations (5) and (6) have been considered, where the lengths and times of the experiment are related.

As can be seen, the gravity calculations closely resemble the theoretical value of 9.81 m/s^2.

For the comparison of the theoretical and practical values, figure 4 was made. The theoretical and experimental data show a practically identical similarity between both.



(a) (b)

Fig. 4. Length as a function of time, (a) theoretical valúes, (b) experimental values.

It was observed that analytical gravity has a value very close to the theoretical value of gravity, while gravity analyzed graphically presents an average error of 0.00011667 to the theoretical value of gravity; this reflects that the experiment was performed consistently and fairly accurately. However, it would be prudent to repeat the tests with a more consistent mass, a thread with less resistance, and a more accurate stopwatch to reduce error values and improve the quality of the process performed m/s^2.

CONCLUSIONS

Throughout the research and the implementation of the theory, it was understood that in many opportunities, the theoretical result is very similar to the practical one. However, it must be taken into account that these values may differ if, when taking the measurements, the forecast of having precision in them needs to be taken.

Statistics is a great tool that facilitates calculations and allows you to predict the results obtained throughout the studies.

On the other hand, it is confirmed that statistics and engineering can shake hands when conducting studies. Statistics is a great ally when doing research because it helps predict results and know if the results being obtained are adequate.

The practical formulations in the understanding of concepts of physics and statistics are beneficial for teaching and learning in engineering careers since they allow interaction with the theoretical context and deepen the concepts achieving more significant learning.

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