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Algorithm for Diffuse TSK Modeling of SNL MIMO with Undefined Operation Points

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Abstract. - This paper presents an algorithm for constructing fuzzy models in linear state subspaces from the nonlinear MIMO dynamic model of plants whose operating points are not defined within the permissible physical range for the system. It is based on the fuzzy Takagi-Sugeno-Kang model and Kawamoto's ideas of non-linearity sectors. The relevant functions in the antecedent are modeled with linear functions, while functions model the consequent in Discrete State Space. The application of the algorithm to the model of a thermoelectric plant widely studied in the specialized literature is discussed.

Keywords: Fuzzy modeling algorithm, undefined points of operation, Kawamoto non-linearity sectors, nonlinear MIMO systems, Takagi-Sugeno-Kang.

Algoritmo para el modelado TSK difuso de SNL MIMO con puntos de operación indefinidos

Resumen: En este artículo se presenta un algoritmo para la construcción de modelos difusos en subespacios de estado lineales a partir del modelo dinámico MIMO no lineal de plantas cuyos puntos de operación – dentro del rango físico permisible para el sistema – no se encuentran definidos. Se toma como base el modelo difuso Takagi-Sugeno-Kang y las ideas de sectores de no linealidad de Kawamoto. Las funciones de pertinencia en el antecedente se modelan con funciones lineales, en tanto que el consecuente se modela mediante funciones en Espacio de Estado Discreto. Se discute la aplicación del algoritmo al modelo de una planta termoeléctrica ampliamente estudiada en la literatura especializada.

Palabras clave: Algoritmo modelado difuso, puntos de operación indefinidos, sectores de no-linealidad Kawamoto, sistemas MIMO no-lineales, Takagi-Sugeno-Kang.

I. INTRODUCTION

To control means to exert the actions necessary to produce a desired result, but to do so, the system to be commanded must exhibit "reasonably predictable" behavior. In the specialized literature [1], it is proposed that the systems to be controlled can be classified into two large groups: deterministic and non-deterministic. Classical control algorithms are based on the hypothesis that the system to be controlled is deterministic, for which a series of restrictions are applied to guarantee the functionality of the algorithm, ranging from restricting the work area to a small region around a point of operation, to "disregarding" the probabilistic nature of the present and future state of the system. There is also a presumption that the mathematical model of the system is time-invariant. However, these classical control techniques work very well for various physical systems and have been successfully employed for over a century, as stated in [2]: "Most physical systems contain complex non-linear relationships, which are difficult to model with conventional techniques." That is why, in advanced process control, non-linear control techniques are used.

One of the ways to mathematically model the non-linear nature of systems is by using models based on fuzzy logic systems. This theory is supported by fuzzy logic systems being universal approximators [2]. In particular, the fuzzy system model developed by Takagi and Sugeno [3] and Sugeno and Kang [4], called the TSK fuzzy model in the literature, is suitable for a broad class of non-linear systems because the consequent is a linear function or even a state-space system. Interestingly, the TSK model allows the use of equations in State Spaces in the consequent, thus being able to obtain a fuzzy model for a Non-linear System (SNL) of multiple inputs and outputs (MIMO), which allows the application of modern control algorithms based on models in State Space such as Optimal Control, H∞, Genetic Algorithms, Predictive Control.

The fuzzy model of an SNL MIMO is robust in applications where the plant has more than one operating point. However, following conventional techniques, as the number of operation points increases, the fuzzy model increases significantly in complexity since, in general, a linear subspace is generated for each operation point, that is, a rule in the consequent. It has also been sufficiently studied that as the number of rules in the consequent increases, the fuzzy TSK model exhibits a behavior closer and closer to the non-linear model of the system. Thus, there is a dilemma between keeping the complexity of the fuzzy model low – few rules – and, at the same time, ensuring that it represents the dynamics of the SNL as accurately as possible.

Now, imagine for a moment that a TSK model is required for an SNL MIMO whose operating points – within the permissible physical range for the system – are not defined, as might be the case with the design of a fuzzy servo controller for such a system. Undoubtedly, this last proposition introduces an additional level of complexity to the previously posed dilemma between keeping the number of rules of the fuzzy model to a minimum and, at the same time, representing the system as accurately as possible, the complexity of not having defined the points of operation. Next, an algorithm is presented to solve the problem: synthesize the fuzzy TSK model of a MIMO SNL with undefined operation points.

II. DEVELOPMENT

In the first instance, a synthesis of the theoretical foundations of the developed algorithm is presented, and then a detailed description is given.

A. *Takagi-Sugeno-Kang Fuzzy Model (TSK)*

Takagi and Sugeno [3], and later Sugeno and Kang [4], developed the structure of a Fuzzy Model that has been widely studied. They denoted the relevance function of a fuzzy set A as *A(x),* with x $\in X$, and defined that all fuzzy sets are associated with linear relevance functions, such that a relevance function is characterized by having two limit values: 1 for the highest degree of relevance and 0 for the lowest degree of significance. Thus, the *Truth Value* of a linguistic proposition of the type "(x is A) Y (y is B)" is expressed as:

$$|"(x\ is\ A)\ and\ (y\ is\ B)"\ | = A(x) \wedge B(y) \qquad (1)$$

In addition, these researchers defined the format of a fuzzy R implication as:

$$R{:}\ If\ f(x_1\ is\ A_1, \ldots, x_k\ is\ A_k), Then\ y = g(x_1, \ldots, x_k) \qquad (2)$$

Where:

y	:	a variable of the consequent whose value is inferred.
$x_1 - x_k$	:	premise variables that also appear in the consequent.
$A_1 - A_k$	:	fuzzy sets with linear relevance functions representing a subspace in which the implication R can be applied by reasoning.
f	:	logical function that relates the propositions in the antecedent.
G	:	a function that involves the value of "y" in the consequent when $x_1 - x_k$ satisfies the antecedent.

If only logical functions are used in the antecedent, and a linear function is adopted in the consequent, implication (2) is written as:

$$R: If\ f(x_1\ is\ A_1\ and\ \ldots\ and\ x_k\ is\ A_k)$$
$$Then\ y = p_0 + p_1x_1 + \ldots + p_kx_k \quad (3)$$

In addition, if i have implications R_i $(i = 1, \cdots, n)$ according to the format indicated in (3). When given:

$$\left(x_1 = x_{10}, \cdots, x_k = x_{k0}\right) \quad (4)$$

where x_{10} - x_{k0} are singletons, the value of y is inferred using the following algorithm:

1) For each implication Ri, y^i is computed by the function g_i of the consequent:

$$y^i = p_0^i + p_1^i x_1^0 + \cdots + p_k^i x_k^0 \quad (5)$$

2) The Truth Value of the proposition y = y^i is calculated by the equation:

$$\left|y = y^i\right| = \left(A_1^i\left(x_{10}\right) \wedge \cdots \wedge A_k^i\left(x_{k0}\right)\right) \quad (6)$$

Where $|*|$ denotes the Truth Value of the proposition, represents the minimum operation, and $\wedge$ A(x_0) is the degree of pertinence of x_0 to the fuzzy set A.

3) Finally, the value of y is inferred from the n implications as its weighted average such that:

$$y = \frac{\sum_{i=1}^{n}\left[\left|y = y^i\right| \times y^i\right]}{\left|y = y^i\right|} \quad (7)$$

B. Kawamoto's Non-Linearity Sectors

According to Mehran [5], the idea of using Non-Linearity Sectors in the construction of fuzzy models was first proposed by Kawamoto [6]. Consider an SNL such that:

$$\dot{x} = f\left(x(t)\right) \quad (8)$$

Where $f(0) = 0$. The aim is to define the global sector in such a way that:

$$\dot{x} = f\left(x(t)\right) \in \left[a_1, a_2\right] x(t) \quad (9)$$

Figure 1 illustrates the nonlinearity sector approach.

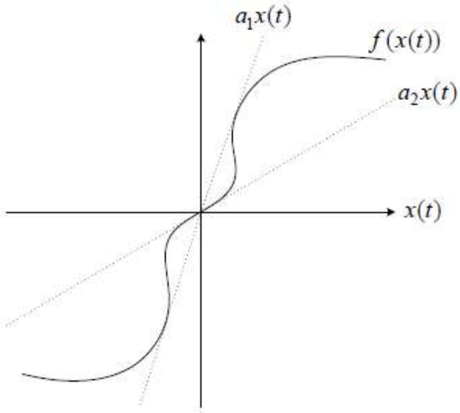

Figure 1. Non-Linearity Sectors.

C. Algorithm for Constructing the TSK Model of a MIMO SNL with Undefined Points of Operation.

Mehran [5] argues that, although the consequent variables can be continuous or discrete in theory, they must be discrete because virtually all fuzzy systems are implemented and modeled using digital systems. Based on the ideas of Takagi, Sugeno, Kawamoto, and Mehran, the following algorithm was designed for the TSK modeling of a MIMO SNL whose operation points are undefined.

Algorithm:

1) From the differential equations of the MIMO SNL, algebraically determine the state-space model for the SNL by considering a generic operation point **(X_0, U_0)**. Such that:

$$\begin{aligned} \dot{\bar{X}}(n+1) &= A(X_0,U_0)\bar{X}(n) + B(X_0,U_0)\bar{U}(n) \\ \bar{Y}(n) &= C(X_0,U_0)\bar{X}(n) + D(X_0,U_0)\bar{U}(n) \end{aligned} \tag{10}$$

2) Analyze the Jacobian matrices *A,* B, *C*, and *D* to determine which variables they depend on. These will be the k premise variables of the TSK model.

3) Analyze the maximum and minimum allowable physical values for the premise variables according to the characteristics of the SNL, or what the maximum and minimum values are for which the SNL model in differential equations is valid. From these, some steady-state pseudo-points of operation of the SNL MIMO are defined and used to obtain the relevance functions of the antecedent and the Jacobian matrices of the consequent. For pseudo-points of operation, the following must be met:

$$0 = A\left(X_0,U_0\right)\bar{X} + B\left(X_0,U_0\right)\bar{U} \tag{11}$$

4) Define the linear relevance functions based on the previous step's results.

5) Define the $n=2^k$ implications of the TSK model according to the following structure:

$$\begin{gathered} R^i: if\,(z_1\ is\ A_1\ and \cdots and\ z_k is A_k) \\ Then\begin{cases} \dot{\bar{X}}(n+1) = A(X_0,U_0)\bar{X}(n) + B(X_0,U_0)\bar{U}(n) \\ \bar{Y}(n) = C(X_0,U_0)\bar{X}(n) + D(X_0,U_0)\bar{U}(n) \end{cases} \end{gathered} \tag{12}$$

6) Infer the value of the output vector from (6) and (7).

III. ALGORITHM AND APPLICATION STUDY

To illustrate the application of the presented algorithm and to verify its effectiveness, the comparison of the results obtained in the modeling of a Thermoelectric Plant widely worked in the specialized literature [7], [8], [9], [10], [11], using both the traditional algorithm based on operating points and the algorithm developed in the research carried out will be presented as a case study.

A. SNL MIMO: Thermoelectric Plant

The model is based on the P16/G16 Thermoelectric Plant at the Sydvenska Kraft AB Plant in Malmö, Switzerland. The power of the Plant is 160 MW, with a Boiler-Turbine-Alternator structure. The 3rd-order model of this SNL MIMO was developed by Bell and Aström [12]. The differential equations that define the dynamics of this MIMO SNL are presented in (13) , (14) and (15):

$$\dot{p} = -0,0018u_2 p^{9/8} + 0,9u_1 - 0,15u_3 \quad (13)$$

$$\dot{P}_o = (0,073u_2 - 0,016)p^{9/8} - 0,1P_o \quad (14)$$

$$\dot{\rho}_f = \frac{\left(141u_3 - \left(1,1u_2 - 0,19\right) p\right)}{85} \quad (15)$$

Where p is the Pressure in the boiler [kg/cm^2], P_o is the electrical power generated [MW], ρ_f is the density of the fluid [kg/m^3], u_1 is the position of the fuel flow valve, u_2 is the position of the steam control valve, and u_3 is the position of the water flow valve. All u_i inputs are normalized in the range [0,1]. Without losing generality, the system outputs are considered the state variables for this study case.

Dimeo and Lee [7] presented the seven points of operation of the Thermoelectric Plant under study, reproduced in Table 1.

Table 1. Thermoelectric Plant Operation Points.

	1	2	3	4	5	6	7
p_0	75,60	86,40	97,20	108,0	118,8	129,6	140,4
P_{o0}	15,27	36,65	50,52	66,65	85,06	105,8	128,9
ρ_{f0}	299,6	342,4	385,2	428,0	470,8	513,6	556,4
u_{10}	0,156	0,209	0,271	0,340	0,418	0,505	0,600
u_{20}	0,483	0,552	0,621	0,690	0,759	0,828	0,897
u_{30}	0,183	0,256	0,340	0,435	0,543	0,663	0,793

B. Discrete TSK Model Based on Points of Operation

To construct the Discrete TSK model Based on the Operating Points of the thermoelectric plant, in the first instance, seven linear subspaces are defined – one for each point of operation indicated in Table 1, then the triangular relevance functions for each of the state variables are described in the antecedent as shown in Figures 2, 3 and 4. Thus, for a given linear subspace, the Truth Value of the proposition, according to (6), is provided by the expression:

$$\left|X = X^i\right| = \left(A_1^i\left(x_{1_0}\right) \wedge A_2^i\left(x_{2_0}\right) \wedge A_3^i\left(x_{3_0}\right)\right) \quad (16)$$

In the case of the consequent, we work with discrete state space models around each point of operation presented in Table 1. Linearized models are obtained from the truncated Taylor series expansion of the SNL represented by the equations(13), (14) and (15). To do this, it is necessary to calculate the following Jacobian matrices:

$$A = \left.\frac{\delta F}{\delta X}\right|_{(X_0,U_0)} \quad (17)$$

$$B = \left.\frac{\delta F}{\delta U}\right|_{(X_0,U_0)} \quad (18)$$

So, the linear approximation of the system will be:

$$\dot{\bar{X}} = A\bar{X} + B\bar{U} \quad (19)$$

where:

$$\bar{X} = X - X^0, X = \left[p \; P_o \; \rho_f\right]^T = \left[x_1 \; x_2 \; x_3\right]^T \quad (20)$$

$$\bar{U} = U - U^0, U = \left[u_1 \; u_2 \; u_3\right]^T \quad (21)$$

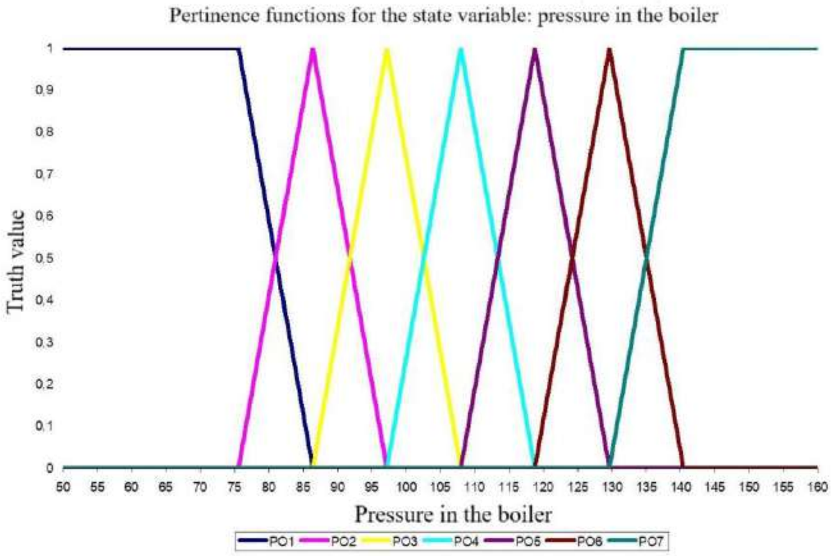

Figure 2. Relevance Functions of the Traditional Discrete TSK Model for Boiler Pressure.

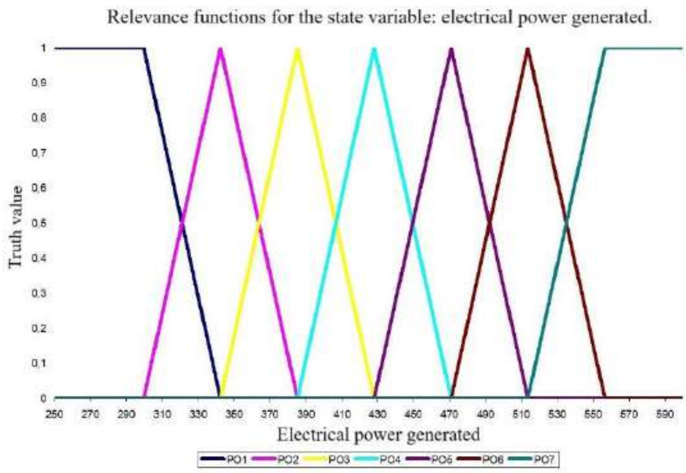

Figure 3. Relevance Functions of the Traditional Discrete TSK Model for Generated Electrical Power.

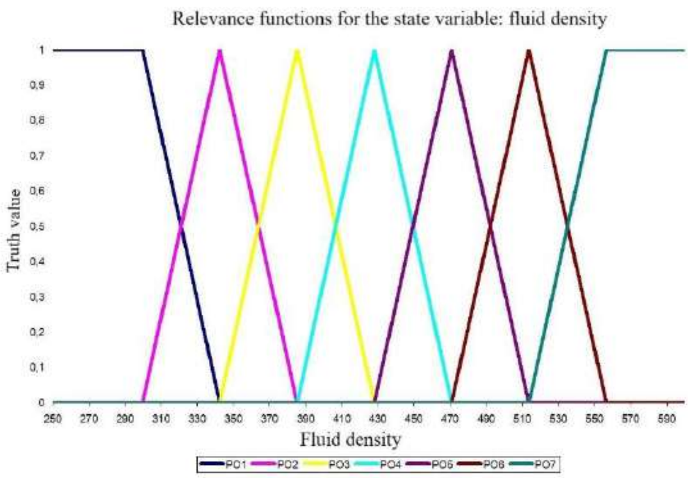

Figure 4. Relevance Functions of the Traditional Discrete TSK Model for Fluid Density.

It was subsequently, using a zero-order retainer-type conversion algorithm, with a sampling time of 60 seconds, the seven pairs of matrices presented from (22) to (28).

$$A_d^{\,1}=\begin{bmatrix}0,9355 & 0 & 0\\ 0,1416 & 0,0025 & 0\\ -0,1172 & 0 & 1,0000\end{bmatrix},\ B_d^{\,1}=\begin{bmatrix}73,1364 & -14,6469 & -12,1894\\ 9,1908 & 100,2430 & -1,5318\\ -4,4779 & -64,3485 & 140,0875\end{bmatrix} \quad (22)$$

$$A_d^{\,2}=\begin{bmatrix}0,9237 & 0 & 0\\ 0,2121 & 0,0025 & 0\\ -0,1549 & 0 & 1,0000\end{bmatrix},\ B_d^{\,2}=\begin{bmatrix}73,6787 & -18,8617 & -12,1131\\ 13,8405 & 128,6948 & -2,3067\\ -5,9344 & -80,6090 & 140,3303\end{bmatrix} \quad (23)$$

$$A_d^{\,3}=\begin{bmatrix}0,8895 & 0 & 0\\ 0,4317 & 0,0025 & 0\\ -0,2779 & 0 & 1,0000\end{bmatrix},\ B_d^{\,3}=\begin{bmatrix}71,3414 & -21,5197 & -11,8902\\ 28,6071 & 145,1289 & -4,7678\\ -10,7098 & -90,6696 & 141,1262\end{bmatrix} \quad (24)$$

$$A_d^{\,4}=\begin{bmatrix}0,8749 & 0 & 0\\ 0,5217 & 0,0025 & 0\\ -0,3258 & 0 & 1,0000\end{bmatrix},\ B_d^{\,4}=\begin{bmatrix}70,7640 & -24,3791 & -11,7940\\ 34,8055 & 163,6185 & -5,8009\\ -12,5898 & -101,3357 & 141,4395\end{bmatrix} \quad (25)$$

$$A_d^{\,5}=\begin{bmatrix}0,8603 & 0 & 0\\ 0,6108 & 0,0025 & 0\\ -0,3729 & 0 & 1,0000\end{bmatrix},\ B_d^{\,5}=\begin{bmatrix}70,1853 & -27,2193 & -11,6975\\ 41,0290 & 181,7735 & -6,8382\\ -14,4483 & -111,7990 & 141,7493\end{bmatrix} \quad (26)$$

$$A_d^{\,6}=\begin{bmatrix}0,8457 & 0 & 0\\ 0,6988 & 0,0025 & 0\\ -0,4191 & 0 & 1,0000\end{bmatrix},\ B_d^{\,6}=\begin{bmatrix}69,6057 & -30,0499 & -11,6009\\ 47,2645 & 199,6554 & -7,8774\\ -16,2853 & -122,1120 & 142,0554\end{bmatrix} \quad (27)$$

$$A_d^{\,7}=\begin{bmatrix}0,8313 & 0 & 0\\ 0,7854 & 0,0025 & 0\\ -0,4646 & 0 & 1,0000\end{bmatrix},\ B_d^{\,7}=\begin{bmatrix}69,0260 & -32,8642 & -11,5043\\ 53,4991 & 217,2193 & -8,9165\\ -18,1007 & -132,2649 & 142,3580\end{bmatrix} \quad (28)$$

Finally, the defusification is performed using the equations (6) and (7).

C. TSK Discrete Model for Undefined Points of Operation

The following is the development of the proposed algorithm to obtain the TSK Discrete model of the thermoelectric plant.

1) The differential equations of the SNL MIMO presented in (13), (14) and (15), are linearized using (17) and (18) around a generic point of operation (X_0, U_0), yielding:

$$\dot{\bar{X}} = A\left(X_0,U_0\right)\bar{X} + B(X_0,U_0)\bar{U} \quad (29)$$

$$A\left(X_0,U_0\right)=\begin{bmatrix} -0,002025u_{2_0}\left(x_{1_0}\right)^{1/8} & 0 & 0\\ \left(0,082125u_{2_0}-0,018\right)\left(x_{1_0}\right)^{1/8} & -0,1 & 0\\ -0,01294118u_{2_0}+0,002235294 & 0 & 0\end{bmatrix} \quad (30)$$

$$B\left(X_0,U_0\right)=\begin{bmatrix} 0,9 & -0,0018\left(x_{1_0}\right)^{9/8} & -0,15\\ 0 & 0,073\left(x_{1_0}\right)^{9/8} & 0\\ 0 & -0,01294118x_{1_0} & 1,658824\end{bmatrix} \quad (31)$$

2) Analyzing (30) and (31), it is concluded that the Jacobian matrices of the linearized system only depend on the variables x_1 and U_2, so these become the premise variables of the TSK model (*k=2).*
3) By analyzing the maximum and minimum physical values of the premise variables x_1 and U_2 and considering that the condition is met (11), the pseudo-points of operation presented in Table 2 are obtained.

Table 2. Pseudo-Operation Points of the Thermoelectric plant.

	T_o	B	c	d
x_{10}	60,02	60,02	165,00	165,00
x_{20}	8,03	51,32	25,02	159,90
x_{30}	275,00	165,80	462,30	474,20
u_{10}	0,0780	0,2427	0,2388	0,7358
u_{20}	0,3287	0,9208	0,3287	0,9208
u_{30}	0,0730	0,3502	0,2008	0,9630

From Table 2, note that for x_1 (Pressure), 60.02 [kgf/cm^2] and 165.00 [kgf/cm^2] have been defined as the minimum and maximum values, respectively, for this premise variable, while for the position of the steam control valve, U_2 0.3287 and 0.9208 have been defined as their maximum and minimum values; from these and from (11), (13), (14) and (15), the rest of the data contained in Table 2 were obtained.

4) Based on the results presented in Table 2, the relevance functions shown in Figures 5 and 6 are defined.

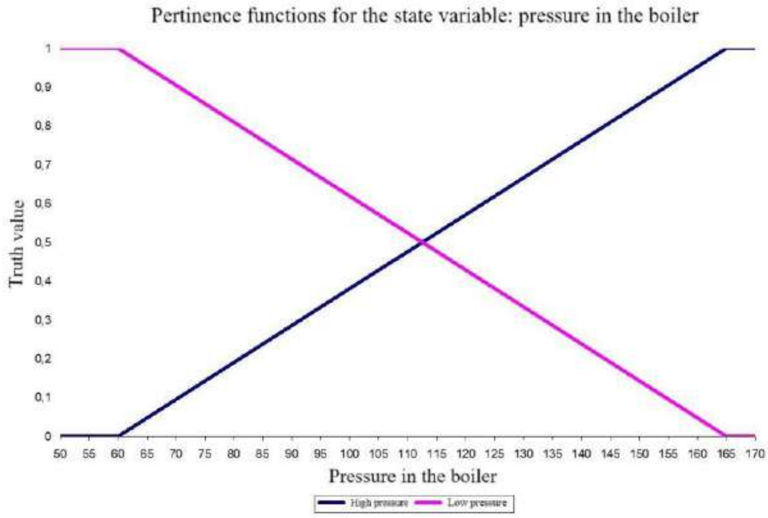

Figure 5. Relevance Functions of the Proposed Discrete TSK Model for Boiler Pressure.

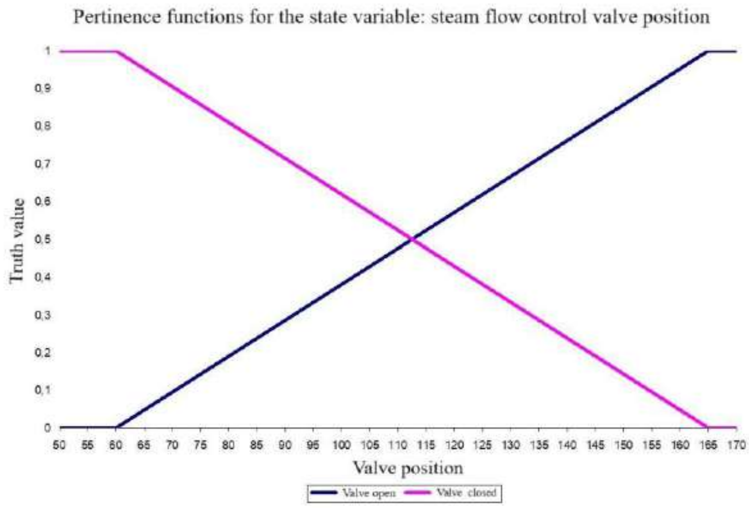

Figure 6. Relevance Functions of the Proposed Discrete TSK Model for Steam Control Valve Position.

Figure 6. Relevance Functions of the Proposed Discrete TSK Model for Steam Control Valve Position.

5) The n=22 implications of the Discrete TSK model is defined:

$$R_1: if\,(x_1\ is\ "low" \wedge u_2\ is\ "closed")$$
$$Then: \bar{X}_1(n+1) = A_1\bar{X}_1(n) + B_1\bar{U}_1(n) \quad (32)$$

Where:

$$A_1 = \begin{bmatrix} 0{,}9355 & 0 & 0 \\ 0{,}1416 & 0{,}0025 & 0 \\ -0{,}1172 & 0 & 1{,}0000 \end{bmatrix}, B_1 = \begin{bmatrix} 73{,}1364 & -14{,}6469 & -12{,}1894 \\ 9{,}1908 & 100{,}2430 & -1{,}5318 \\ -4{,}4779 & -64{,}3485 & 140{,}0875 \end{bmatrix} \quad (33)$$

$$R_2: if\,(x_1 is\ "low" \wedge u_2\ is\ "opened")$$
$$Then: \bar{X}_2(n+1) = A_2\bar{X}_2(n) + B_2\bar{U}_2(n) \quad (34)$$

Where:

$$A_2 = \begin{bmatrix} 0{,}8297 & 0 & 0 \\ 0{,}8208 & 0{,}0025 & 0 \\ -0{,}5299 & 0 & 1{,}0000 \end{bmatrix}, B_2 = \begin{bmatrix} 68{,}9638 & -13{,}8113 & -11{,}4940 \\ 55{,}9540 & 90{,}8778 & -9{,}3257 \\ -20{,}6518 & -61{,}1094 & 142{,}7832 \end{bmatrix} \quad (35)$$

$$R_3: if\,(x_1 is\ "high" \wedge u_2\ is\ "closed")$$
$$Then: \bar{X}_3(n+1) = A_3\bar{X}_3(n) + B_3\bar{U}_3(n) \quad (36)$$

Where:

$$A_3 = \begin{bmatrix} 0{,}9272 & 0 & 0 \\ 0{,}1595 & 0{,}0025 & 0 \\ -0{,}1166 & 0 & 1{,}0000 \end{bmatrix}, B_3 = \begin{bmatrix} 72{,}8127 & -45{,}4890 & -12{,}1355 \\ 10{,}3892 & 311{,}9603 & -1{,}7315 \\ -4{,}4647 & -176{,}5755 & 140{,}0853 \end{bmatrix} \quad (37)$$

$$R_4: if\,(x_1 is\ "high" \wedge u_2\ is\ "opened")$$
$$Then: \bar{X}_4(n+1) = A_4\bar{X}_4(n) + B_4\bar{U}_4(n) \quad (38)$$

Where:

$$A_4 = \begin{bmatrix} 0{,}8091 & 0 & 0 \\ 0{,}9121 & 0{,}0025 & 0 \\ -0{,}5235 & 0 & 1{,}0000 \end{bmatrix}, B_4 = \begin{bmatrix} 68{,}1305 & -42{,}5638 & -11{,}3551 \\ 62{,}8271 & 279{,}2003 & -10{,}4712 \\ -20{,}4850 & -166{,}5670 & 142{,}7554 \end{bmatrix} \quad (39)$$

In general:

$$\bar{X}_i(n) = X(n) - X_{i_0}\ ,\ \bar{U}_i(n) = U(n) - U_{i_0} \quad (40)$$

7) The value of the output is inferred from (6) and (7).

IV. RESULTS

Two comparative studies were conducted to show the performance of the Discrete TSK model developed with the proposed algorithm. The first study consisted of subjecting the three models (the non-linear, the Fuzzy model with seven rules, and the Fuzzy model formulated with the proposed algorithm) to a sequence of ascending and descending inputs according to the seven points of operation presented in Table 1. The second comparative study involved subjecting the three models to a pseudo-random sequence of the three input variables, respecting the established limits (minimum and maximum values) for each input in Table 1.

For both comparative studies, Variation Accounting (VAF) and Mean Square Error (RMSE) were considered as performance indices, indices used by Castillo, Sarmiento, and Sanz [13] when performing the comparative evaluation of a similar discrete TSK model. Recall that as two signals in a time series are almost identical, the RMSE tends to zero, while the VAF tends to 100%.

Figures 7 to 10 present the most relevant results of the first comparative study, while Figures 11 to 14 present the results of the second study.

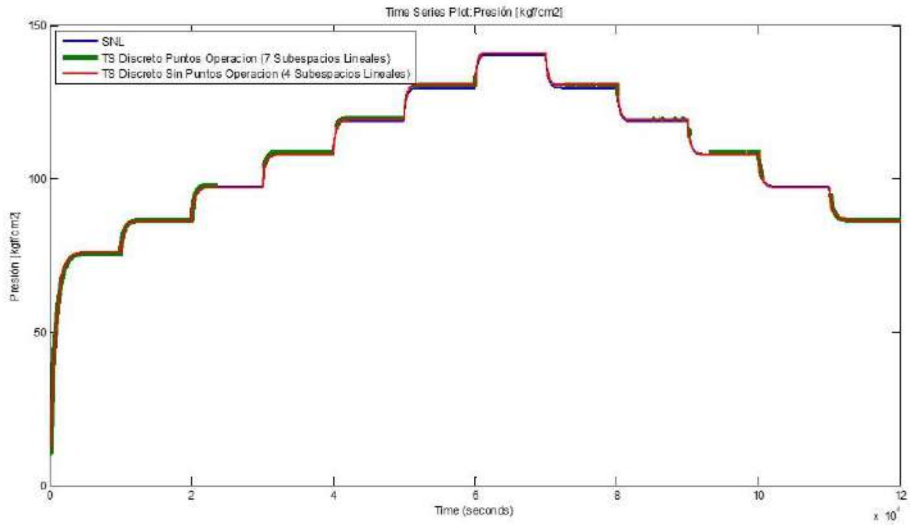

Figure 7. Comparison of the estimation of the Pressure in the Boiler of the Thermoelectric Plant for case 1.

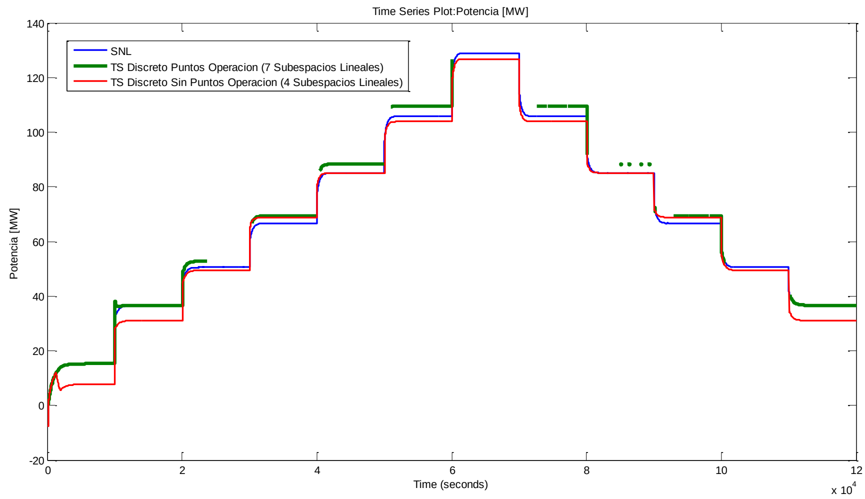

Figure 8. Comparison of the estimation of the Power Output of the Thermoelectric Plant for case 1.

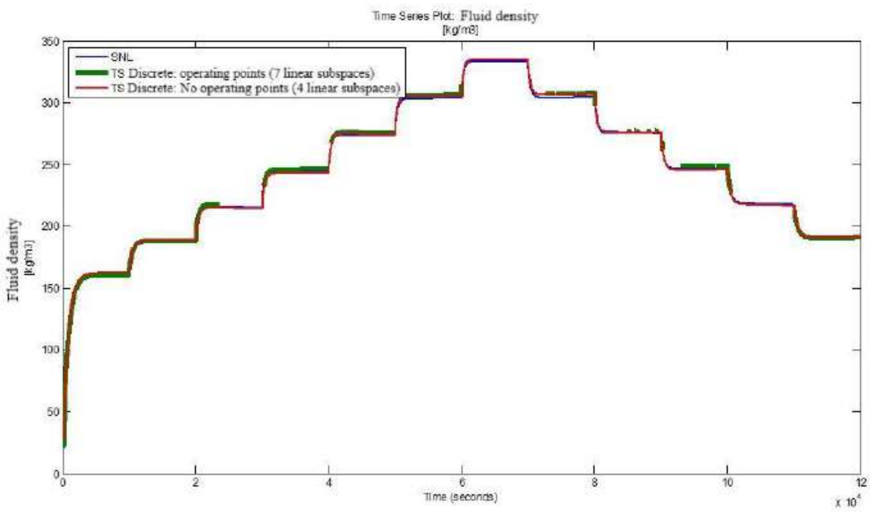

Figure 9. Comparison of the Fluid Density estimation in the Thermoelectric Plant for Case 1.

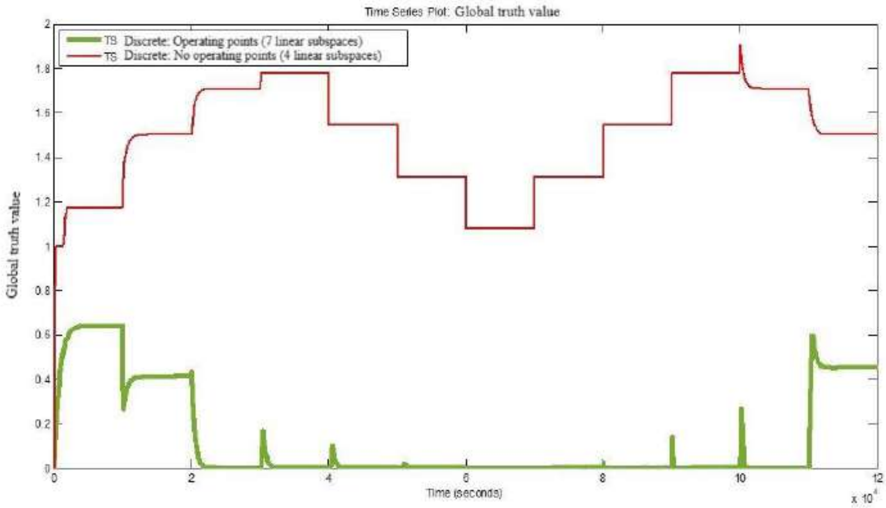

Figure 10. Comparison of the Truth Value of the TSK Models for Case 1.

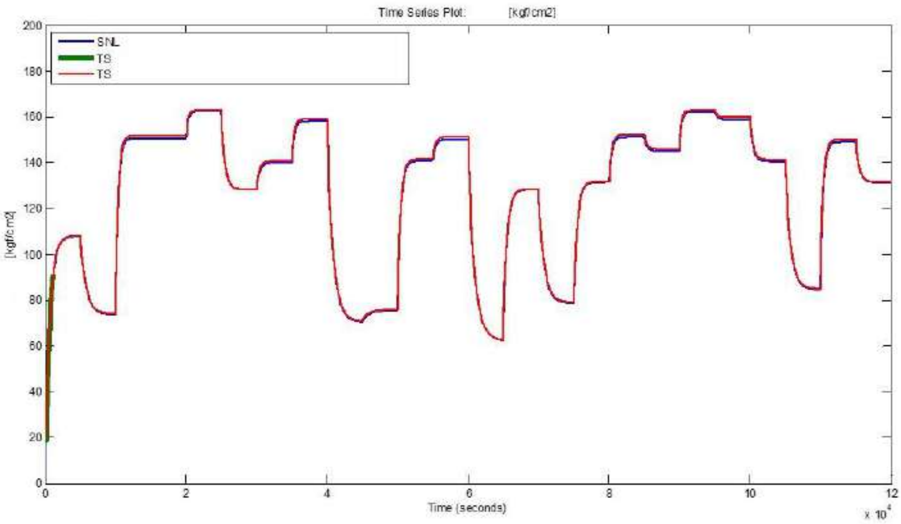

Figure 11. Comparison of the estimation of the Pressure in the Boiler of the Thermoelectric Plant for case 2.

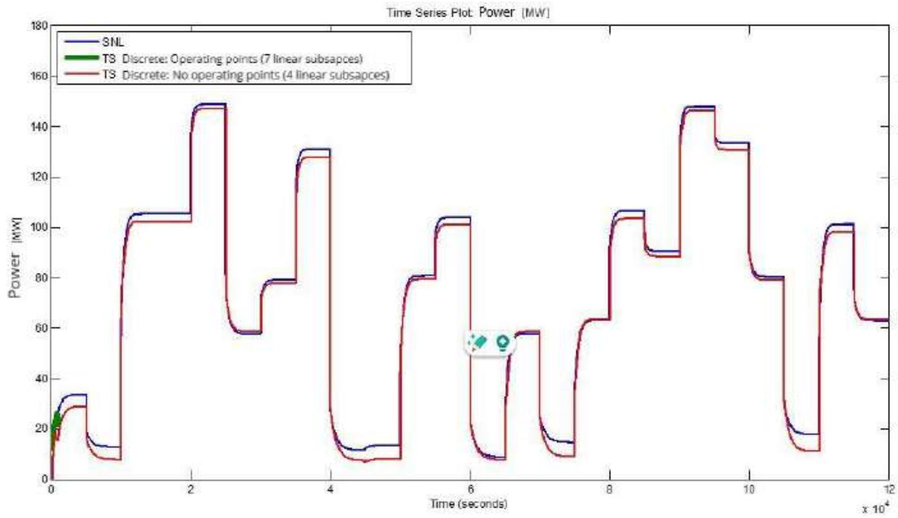

Figure 12. Comparison of the Output Power estimation of the Thermoelectric Plant for case 2.

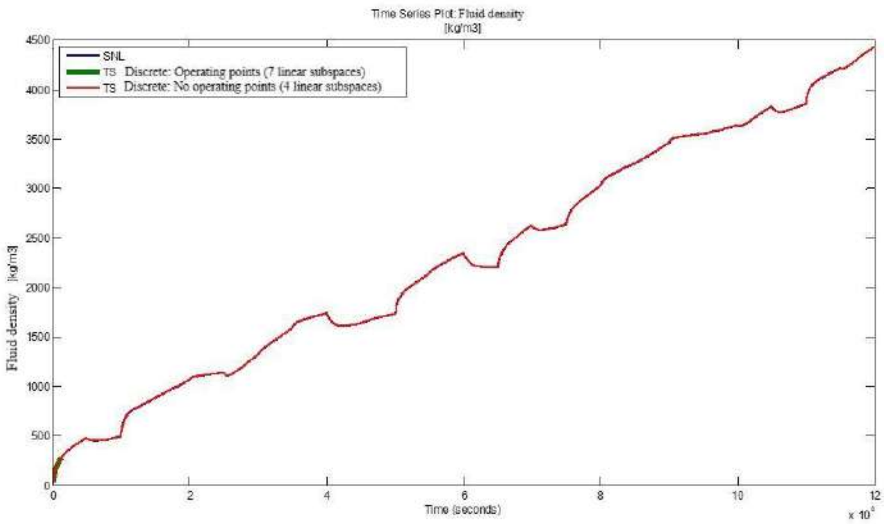

Figure 13. Comparison of the Fluid Density estimation in the Thermoelectric Plant for Case 2.

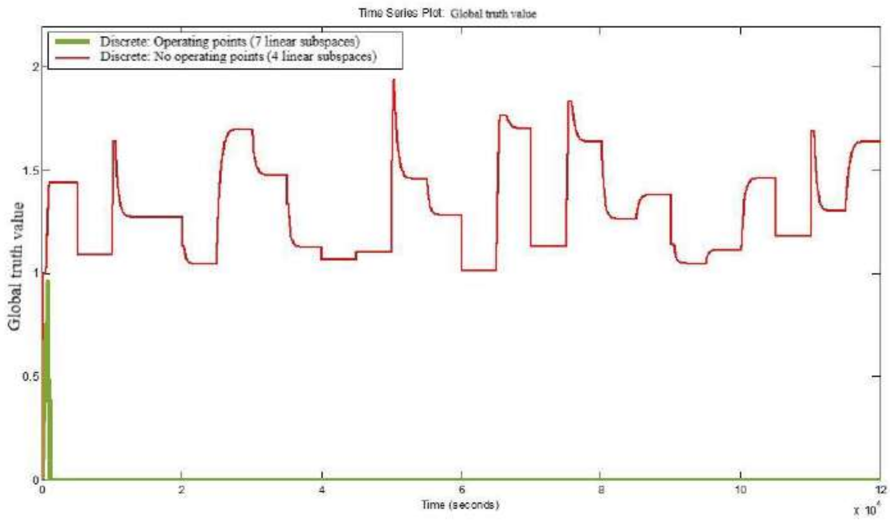

Figure 14. Comparison of the Truth Value of the TSK Models for Case 2.

Table 3 shows the results of the measurement of performance indices for each study.

Table 3. TSK Model Performance Indices.

	TSK Model – 7 Rules				TSK Model – 4 Rules			
	Case 1		Case 2		Case 1		Case 2	
	VAF%	RMSE	VAF	RMSE	VAF%	RMSE	VAF%	RMSE
x_1	Nan	Nan	Nan	Nan	99,96	0,51	99,93	5,08
x_2	Nan	Nan	Nan	Nan	99,05	3,63	97,58	7,19
x_3	Nan	Nan	Nan	Nan	99,94	1,40	100,00	2,80

Table 4 presents the percentage error of the 4-rule TSK model implemented with the proposed algorithm with respect to the nonlinear model, both for case study 1 and case 2.

Table 4. TSK Model Error – 4 Rules Regarding the Non-linear Model.

	TSK Model – 4 Rules	
	Case 1	Case 2
	%Error	%Error
x_1	0.04%	0.07%
x_2	0.95%	2.42%
x_3	0.06%	0.00%

A. *Discussion of the Results*

For case study 1 (sequence of ascending and descending inputs), in Figures 7, 8, and 9, it is evident that the Discrete TSK model obtained from the points of operation (7 Rules) is not able to follow the sequence of inputs continuously, this is corroborated when analyzing Figure 10, where it is shown that the Global Truth Value of the model becomes zero at multiple points in the trajectory. This cancels the output of the TSK model at those intervals; as a consequence of this discontinuity, it is impossible to calculate the VAF and RMSE performance indices for this TSK Model in case study 1, as shown in Table 3. In contrast, the Discrete TSK Model obtained from the proposed algorithm, if it was able to follow the trajectory imposed by the sequence of inputs, is corroborated in Figures 7, 8, and 9 (red line) and even in Figure 10, in which the Global Truth Value of the model is never canceled. This makes it possible to calculate the VAF and RMSE performance indices, which were above 99.00% and below 3.7 respectively, values that are very close to the ideal case (VAF=100% and RMSE=0).

For case study 2 (sequence of pseudo-random inputs), in Figures 11, 12, and 13, it is again evident that the Discrete TSK model obtained from the points of operation is not able to follow the sequence of inputs, even the performance is still lower than case 1, which is corroborated when analyzing Figure 14 and comparing it with Figure 11; again as a consequence of such discontinuity, it is not possible to calculate the VAF and RMSE performance indices for this TSK Model in case study 2, as shown in Table 3. In contrast, the Discrete TSK Model obtained from the proposed algorithm was once again able to follow the trajectory imposed by the sequence of inputs, which are even more demanding (because they are random), which is corroborated in Figures 11, 12 and 13 (red line), and even in Figure 14, in which the Global Truth Value for this model is clearly never cancelled; As for the VAF and RMSE performance indices, which were above 97.50% and below 7.20 respectively, results that, although they show lower performance than that obtained in case 1, turn out to be extremely interesting given the rigorous sequences of inputs to which the model was subjected.

For both cases, the discrete TSK model obtained from the proposed algorithm was able to follow the SNL satisfactorily.

CONCLUSIONS

In this paper, we have presented an algorithm to synthesize the fuzzy discrete TSK model in linear state subspaces for a MIMO SNL, from the dynamical model in differential equations, without the SNL operating points having been previously defined. The relevance functions in the antecedent are modeled with linear functions. The application of the algorithm to the model of a Thermoelectric Power Plant has been discussed, widely studied in the specialized literature, obtaining satisfactory values in the chosen performance indices (VAF and RMSE). It is expected that this methodology will serve to promote the application of modern trajectory tracking control algorithms based on models in State Space such as: Optimal Control, H∞, Genetic Algorithms, Predictive Control.

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