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Novel methodology for characterization of thermoelectric modules and materials

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Abstract. - The document presents an innovative methodology that combines forced response and natural response theories in thermoelectric materials and devices. It stands out for expressing the thermoelectric figure of merit in terms of the ratio of two temperatures $Z\bar{T} = \Delta T'/\Delta T$, enabling comprehensive testing and precise characterization of thermoelectric modules and materials, including measurements of thermal conductance, electrical resistance, Seebeck coefficient, and figure of merit. Additionally, it addresses the determination of thermal resistances and thermal capacitances related to thermal contacts, as well as the derivation of characteristic time constants and angular frequencies. This approach, applicable to both modular devices and individual samples, allows for the simultaneous measurement of all parameters on a single sample. The experiments considered non-ideal contacts and non-adiabatic conditions at room temperature $\bar{T} = 300K$, enhancing the feasibility of in-situ characterization and positioning this methodology as a key tool in thermoelectric research.

Keywords: thermoelectric characterization, Harman method, transient test method, thermoelectric time constants, thermoelectric frequencies, complete response, figure of merit.

Metodología novedosa para la caracterización de módulos y materiales termoeléctricos

Resumen: El documento presenta una metodología innovadora que combina teorías de respuesta forzada y respuesta natural en materiales y dispositivos termoeléctricos. Destaca por expresar la figura de mérito termoeléctrica en términos de la relación de dos temperaturas $Z\bar{T} = \Delta T'/\Delta T$, permitiendo ensayos completos y caracterizaciones precisas de módulos y materiales termoeléctricos, incluyendo mediciones de conductancia térmica, resistencia eléctrica, coeficiente de Seebeck y figura de mérito. Además, aborda la obtención de resistencias térmicas y capacitancias térmicas relacionadas con contactos térmicos, así como la determinación de constantes de tiempo características y frecuencias angulares. Este enfoque, aplicable tanto a dispositivos modulares como a muestras individuales, posibilita la medición simultánea de todos los parámetros en una misma muestra. Los experimentos consideraron contactos no ideales y condiciones no adiabáticas a temperatura ambiente *T=300K* mejorando la viabilidad de la caracterización in situ y posicionando esta metodología como una herramienta clave en la investigación termoeléctrica.

Palabras clave: caracterización termoeléctrica, método de Harman, método de prueba transitorio, constantes de tiempo termoeléctricas, frecuencias angulares termoeléctricas, figura de mérito.

I. INTRODUCTION

The Thermoelectric Modules and Thermoelectric Materials (TEMs) are solid-state power converters that typically consist of p and n type semiconductor material arrangements, connected so that they are thermally in parallel and electrically in series [1] [2] [3] [4]. TEMs are marketed for applications in the areas of cooling and heating; as well as for the generation of electrical power, collecting solar energy and residual heat [5] [6]. One of the arduous tasks that must be carried out in the thermoelectric field is the characterization of thermoelectric materials and modules [7]. It is essential to obtain reliable measurements of global efficiency to assess its technological and economic interest [8] [9]. Another challenge to meet is to determine the performance of thermoelectric devices adequately and accurately [10]. The thermoelectric performance is reduced to the determination of a single quantity called the figure of merit $Z\bar{T}$ and a way of expressing it is presented in (1), where $\bar{T}$ is the average working temperature of the system, R_m is the electrical resistance of the module, K_0 is the thermal conductance to the vanishing electric current, and α is the global Seebeck coefficient characterizing the thermoelectric coupling between the electric current and the heat flux through the TEM terminals. The figure of merit $Z\bar{T}$ is related to the theoretical maximum efficiency of a thermoelectric generator that works between two thermal reservoirs, one at a hot temperature T_h and the other at a cold temperature T_c, where $T_c < T_h$. The maximum efficiency η_{max} is determined by means of (2), in which $\eta_C = 1 - T_c/T_h$ is the Carnot efficiency [11] [12].

$$Z\bar{T} = \frac{\alpha^2 \bar{T}}{R_m K_0} \quad (1)$$

$$\eta_{max} = \eta_C \frac{\sqrt{1 + Z\bar{T}} - 1}{\sqrt{1 + Z\bar{T}} + T_c/T_h} \quad (2)$$

The precise evaluation of $Z\bar{T}$ is far from simple, and several approaches can be applied, e.g.: measure α, K_0 y R_m separately and then calculate $Z\bar{T}$ using (1). However, this method is quite inaccurate without great experimental attention, since each measurement error for each parameter contributes to the accumulated global error in the resulting $Z\bar{T}$ value [13] [14]. Other methods use different measurement systems for each property. Often the three main properties are not measured on the same sample or in the same direction. Also, these methods are time-consuming and susceptible to increasing the uncertainties in $Z\bar{T}$. In the late 1950's a method was presented by Harman for testing the resistance in alternating current and determining the figure of merit of a thermoelectric material sample [15]. Also, the same methodology was used by Harman, Cahn, and Logan for the measurement of thermal conductivity applying the Peltier Effect [16]. This technique has many variations and it has been traditionally applied to both bulk modules and thin films. The drawbacks are that it only works with small temperature differences and requires adiabatic boundary conditions that can be difficult to satisfy [17] [18]. In the early 1990s a test methodology was developed by Buist, this method is referred to as the transient test method [19], which is based on a similar concept but some fundamental differences compared with Harman's method which gave rise to improvement in accuracy, and reproducibility [5] [20]. The fundamental similarity is that the two techniques cited above are designed to solve the voltage components of a thermoelectric device and the fundamental difference is that the Harman method does this by measuring the resistive component and the transient test method measures the Seebeck component, whereas the novel methodology for characterization hat this moment described is designed by determination of the thermal components, and it is intended to provide the ultimate solution for measuring the thermal conductance, thermal conductivity and the figure of merit of TEMs.

The new method presented in this paper provides a set of guidelines for the direct measurement of all the parameters needed to characterize the thermoelectric properties of either materials and devices under test, such as the thermoelectric resistance of the module R_{TE}, thermal resistances related to thermal contacts R_C, thermoelectric capacitance of the module C_{TE}, thermal capacitances related to thermal contacts C_C, thermal resistance of the thermoelectric material R_0 and the capacitance of the thermoelectric material C_{th}. As well the Seebeck coefficient α, electrical resistance R_m, thermal conductance K_0, electrical resistivity ρ, thermal conductivity κ and figure of merit $Z\bar{T}$. Additionally, through this methodology is possible to determine the characteristic time constants and relaxation times, as well as the characteristic angular frequencies.

This article is structured as follows: in section II the complete response of TEMs is presented, then in section III the temperature and voltage stability are explained, obtaining as a result the characteristic thermoelectric time constants τ_{TE}, τ_{th}, τ_C and the relaxation times, and it continues to section IV developing thermoelectric modeling and equation derivations, where new equations of the figure of merit $Z\bar{T}$ are shown in V characterization configurations. Finally, the experimental results, implementation, conclusions, and references. Furthermore, this research is framed within UNESCO's 2015-2030 agenda for sustainable development, specifically objective number 7, entitled “Affordable and Clean Energy”, which aims to improve access to clean energy through inclusive science, technology, and innovation systems (STI).

II. COMPLETE RESPONSE OF THERMOELECTRIC MODULES AND MATERIALS

The temperature difference and voltage of TEMs generated due to the Peltier and Seebeck effects, respectively [2] [1], have two components and there are two classical ways to break it down into two parts. The first way is to divide it into “a forced response (independent source) and a natural response (stored energy)”, and the second way is to divide it into “a steady-state response (permanent or stable part, this is the behavior of the TEMs long after external excitation applied) and a transient response (temporary part, which will extinguish with time)” [20].

The unification of the forced response (slow and fast perturbation) and natural response (absence of perturbation) theories allows for a description of the complete response of thermoelectric modules and materials; as well as allows the study and characterization of TEMs [21] [22] [23]. The equations used to represent the complete response (or total response) of a thermoelectric module either to the abrupt application of a DC voltage source on the electrical terminals or to the abrupt application of a temperature differential on the thermal contacts are shown in (3) and (4), respectively; assuming that the thermoelectric module represents a thermoelectric capacitor initially discharged and at $t = 0$, $\Delta T(0) = 0$, and $V_\alpha(0) = 0$.

$$\Delta T(t) = \alpha\bar{T}IR_{TE} + [\Delta T(0) - \alpha\bar{T}IR_{TE}]e^{-t/\tau_{TE}} \quad (3)$$
$$V_\alpha(t) = \alpha\Delta T + [V_\alpha(0) - \alpha\Delta T]e^{-t/\tau_{TE}} \quad (4)$$

A. Forced response of thermoelectric modules and materials

The forced response of TEMs can be obtained through the temperature difference or the voltage generated by a TEM, from the corresponding Peltier and Seebeck effects, and are given by the mathematical expressions (5) and (6) [22]. Where $\Delta T(t)$ is the temperature difference measured across the thermal contacts of the TEM, α is the Seebeck coefficient, $\bar{T}$ is the average temperature, R_{TE} is the thermoelectric resistance, I is the electric current. Also, $\Delta T(t) = T_h(t) - T_c(t)$, $\alpha = V_\alpha/\Delta T$ and $I = [V_s u(t) + V_\alpha(t)]/R_m$, in which $V_s u(t)$ represents the external voltage source, $V_\alpha(t)$ the Seebeck voltage, R_m the electrical resistance of the TEM. And τ_{TE} is the thermoelectric time constant of the module.

$$\Delta T(t) = \alpha \bar{T} I R_{TE}\left(1 - e^{-t/\tau_{TE}}\right) \quad (5)$$
$$V_\alpha(t) = \alpha \Delta T\left(1 - e^{-t/\tau_{TE}}\right) \quad (6)$$

Consequently, for the case where the thermal contacts are absent; i.e., the case where there is only the presence of thermoelectric material, mathematical expressions (5) and (6) are reduced to expressions (7) and (8), where $\Delta T'$ is the temperature differential generated at the thermal terminals by the thermoelectric material, R_0 is the thermal resistivity of the thermoelectric material and τ_{th} is the characteristic thermoelectric time constant of the thermoelectric material.

$$\Delta T'(t) = \alpha' \bar{T} I R_0\left(1 - e^{-t/\tau_{th}}\right) \quad (7)$$
$$V'_\alpha(t) = \alpha' \Delta T'\left(1 - e^{-t/\tau_{th}}\right) \quad (8)$$

It should be noted that the forced response of TEMs expressed in the temperature difference is the exponential increase of the Peltier effect and expressed in voltage is an exponential increase of the Seebeck effect and complies with the theory of first-order electrical circuits [22].

B. Natural response of thermoelectric modules and materials

Since the theory of the natural response of TEMs, it is known that the temperature difference associated with the thermoelectric capacitance of the thermoelectric material is $\Delta T_{C_{th}}$ and is defined as $\Delta T_{C_{th}} = \Delta T' = T_{hM} - T_{cM}$ and the temperature difference related to the equivalent thermal capacitance of the thermal contacts is given by ΔT_{C_C}. Thus, the natural response of a thermoelectric module is defined by equations (9) and (10) [23].

$$\Delta T = \Delta T' + \Delta T_{C_C} \quad (9)$$
$$V_\alpha = \alpha\left[\Delta T' + \Delta T_{C_C}\right] \quad (10)$$

The formal mathematical expression for ΔT_{C_C} is presented in (11).

$$\Delta T_{C_C} = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (11)$$

Where, $s_1 = -\beta + \sqrt{\beta^2 - \omega_0{}^2}$ and $s_2 = -\beta - \sqrt{\beta^2 - \omega_0{}^2}$ are the roots and are called natural frequencies, measured in Nepers per second (Np/s), because they are associated with the natural response of the thermoelectric device; ω_o is called the resonant frequency of the thermoelectric module, or more strictly the undamped natural frequency of the TEM, expressed in radians per second (rad/s), and β is the natural frequency or damping factor, expressed in Nepers per second, and represents the frequency associated with the thermal contacts and is also called ω_1, therefore $\beta = \omega_1$. Further, considering that at $t = 0$, $A_1 = \alpha \bar{T} I_{TEG_{sc}} R_{c_{cold}}$ and $A_2 = \alpha \bar{T} I_{TEG_{sc}} R_{c_{hot}}$, where $R_{c_{cold}}$ is the thermal resistance of the contact cold side of the TEM, $R_{c_{hot}}$ is the thermal contact resistance of the hot side of the TEM, and $I_{TEG_{sc}}$ is the short-circuit current of the device in thermo generator mode; that is, the load R_{Load} connected to the thermogenerator is zero ohms. So, for $R_{Load} = 0$, we have that $I_{TEG_{sc}} = V_\alpha / R_m$. And the formal mathematical expression for $\Delta T'(t)$ is as shown in (12).

$$\Delta T'(t) = A_1 e^{-\beta_{th} t} \cos \omega_d t + A_2 e^{-\beta_{th} t} \operatorname{sen} \omega_d t \quad (12)$$

Therefore, the natural response concerning the thermoelectric material is defined using the following expressions (13) and (14).

$$\Delta T'(t) = e^{-\beta_{th} t}(A_1 \cos \omega_d t + A_2 \operatorname{sen} \omega_d t) \quad (13)$$
$$V'_\alpha(t) = \alpha'\left[e^{-\beta_{th} t}(A_1 \cos \omega_d t + A_2 \operatorname{sen} \omega_d t)\right] \quad (14)$$

The time constant associated with the capacitance of the thermoelectric material τ_{th} could be obtained at a slightly longer time than expected. Considering that the amplitude is reduced by a factor $1/e$ (to 36.8 % of what it had), it can be achieved at a longer time to the relation $\tau_{th} = 1/\omega_{th}$, where ω_{th} is the characteristic angular frequency related to the thermoelectric material, for which the case of an overdamped thermoelectric circuit is considered when $\beta < \omega_0$. Therefore, the roots can be write as $s_1 = -\beta + \sqrt{-({\omega_0}^2 - \beta^2)} = -\beta + j\omega_d$, and $s_2 = -\beta - \sqrt{-({\omega_0}^2 - \beta^2)} = -\beta - j\omega_d$, where $j = \sqrt{-1}$ and $\omega_d = \sqrt{({\omega_0}^2 - \beta^2)}$ is called the damping frequency. Both ω_0 and ω_d are natural frequencies, because they help determine the natural response of the TEM; while ω_o is often called the undamped natural frequency, ω_d is called the damped natural frequency or offset frequency. Such a response has a time constant associated with the thermoelectric capacitance τ_{th} and a period of $T = 2\pi/\omega_d$, and as the amplitude is reduced by a factor $1/e$ (to 36.8 % of what it had), we then have two scales of time: T measures the time it takes to oscillate and τ_{th} the time it takes to damp. The dimensionless quotient is represented in (15) [23].

$$\frac{\tau_{th}}{T} = \frac{\omega_d}{2\pi\beta_{th}} \tag{15}$$

It is important to highlight that the natural response of TEMs expressed in temperature is an exponential drop in the temperature difference and satisfies the Newton's law of cooling, and the natural response of TEMs expressed in voltage is an exponential Seebeck voltage drop and complies with second order electrical circuits theory [23].

C. Temperature and voltage stability

The thermoelectric time constant τ_{TE} is obtained from the forced response (step response) of TEMs, from the temperature difference $\Delta T(t)$ on the faces of the module (Peltier effect), from the temperatures of either any of the thermal contacts $T_c(t)$, $T_h(t)$ or electric potencial $V_\alpha(t)$ generated between the positive and negative terminals of the TEMs (Seebeck effect) [22]. The time constant τ_{TE}, corresponds to the inverse of the characteristic thermoelectric angular frequency of the TEMs $\tau_{TE} = 1/\omega_{TE}$ and is the time required for $\Delta T(t)$, $T_c(t)$, $T_h(t)$ y $V_\alpha(t)$ to increase by a factor of "e" or 63.2% of its final value; that is, they take $5\tau_{TE}$ to reach their steady state, when no change occurs over time [22]. Taking into account that, $\tau_{TE} = C_{TE}R_{TE}$.

The time constant associated with the capacitance of the thermal contacts τ_C is obtained from the natural response of the TEMs, specifically from the temperature of the thermal contact related to the cold side [23]. For the case where the thermal contact of the cold side is equal to the thermal contact of the hot side, the time constant τ_C is obtained experimentally from the temperature measurement on the face of the cold side of the device T_c and is determined considering that the amplitude is increased by a factor $1/e$ (36.8 % of the amplitude that it would have), so $\tau_C = 1/\beta = 1/\omega_1$, where $\tau_C = C_C R_C$. The damping factor β determines the rate at which the response is damped. The time it takes for the temperature of the cold side to rise is given by the decay of factor β. So the thermal capacitor will be fully discharged after five time constants. In other words, the capacitor associated with the thermal contact on the cold side of the TEM takes $5\tau_C$ to reach its final state [23].

The thermoelectric time constant related to the thermoelectric material τ_{th} is also obtained from the natural response. From equation (15) an expression is obtained for the characteristic angular frequency related to the thermoelectric material which is known as ω_{th}, and is given by the following mathematical expression in (16).

$$\omega_{th} = 2\pi\beta_{th} = \frac{2\pi}{\tau_{th}} \tag{16}$$

The time that takes the amplitude of the temperature difference between the faces of the TEMs (either ΔT or $\Delta T'$) to decay is given by the time constant $\tau_{th} = C_{th}R_0$ is obtained through the following equation (17) [23].

D. Thermoelectric modeling and equation derivation

Thermoelectric coefficients and parameters *of TEMs*

Using the theory of forced response, it is possible to obtain the thermoelectric resistance R_{TE}, using (5) [22]. Considering that, for the forced response, at $t = 0$, the temperature difference $\Delta T(0) = 0$ and that after $5\tau_{TE}$ it will reach its steady state, then $R_{TE} = \Delta T(t)/\left[\alpha \bar{T} I \left(1 - e^{-t/\tau_{TE}}\right)\right]$, and it can be expressed in a more compact way if it is taken into consideration that for a time greater than $t > 5\tau_{TE}$, $e^{-t/\tau_{TE}}$ tends to zero, then an expression for R_{TE} is shown in (18).

$$R_{TE} = \frac{\Delta T}{\alpha \bar{T} I} \tag{18}$$

Using the theory of natural response, the equivalent resistance of the thermal contacts is obtained R_C and is found through (11) [23]. Equation (11) shows that ΔT_{C_C} is the result of the temperature contribution of each thermal contact and R_C is the result of the sum of the thermal resistances of the thermal contacts. Assuming that the thermal resistances corresponding to the contacts are equal, $R_{C_{cold}} = R_{C_{hot}}$, that the equivalent resistance of the contacts is $R_C = R_{C_{cold}} + R_{C_{hot}}$ and $A_1 = A_2$, then $\Delta T_{C_C}(t) = \alpha \bar{T} I_{TEG_{sc}} R_C e^{s_1 t}$. Taking into account that, at $t = 0$, the maximum value of ΔT_{C_C} is obtained. Additionally, that $V_\alpha = \alpha \Delta T$ and $I_{TEG_{sc}} = V_\alpha / R_m$, so $I_{TEG_{sc}} = \alpha \Delta T / R_m$. Therefore, the thermal resistance related to thermal contacts can be written as in (19) [22] [23]. The electric resistance of TEMs R_m is found using (20) [3] [4]. And, to find the electric resistivity ρ it is recommended to use the equation (22), which correlates R_m and the geometry of the specimen under test.

$$R_C = \frac{R_m \Delta T_{C_C}}{\alpha^2 \bar{T} \Delta T} \tag{19}$$

$$R_m = \frac{V_{max}}{I_{max}} \frac{(T_h - \Delta T_{max})}{T_h} \tag{20}$$

Also, using the theory of natural response, the thermal conductance K_0 is obtained, considering the equation (12). With the presence of the sine and cosine functions it is trivial that the natural response for this case is exponentially damped and oscillatory in nature. Considering that, at $t = 0$ the maximum value of the temperature difference $\Delta T'$; also that $A_1 = A_2 = \alpha \bar{T} I_{TEG_{sc}} R_0$, $\operatorname{sen} \omega_d t = 0$ and $\cos \omega_d t = 1$; that is, a term in (12) contributes neither to the temperature difference $\Delta T'(t)$ nor to the Seebeck voltage $V_\alpha(t)$. Therefore, the thermal resistance of the thermoelectric material is $R_0 = \Delta T' / \alpha \bar{T} I_{TEG_{sc}}$ and knowing that the thermal conductance is equal to the inverse of the thermal resistance; that is, $K_0 = 1/R_0$, then the thermal conductance of the thermometric material is given by $K_0 = \alpha \bar{T} I_{TEG_{sc}} / \Delta T'$. Hence, the thermal conductance of the thermoelectric material can be written as in (21), keeping in mind that $I_{TEG_{sc}} = \alpha \Delta T / R_m$. Furthermore, from the forced response is possible to find that $K_0 = (1/R_{TE}) - (1/R_C)$.

$$K_0 = \frac{\alpha^2 \bar{T} \Delta T}{R_m \Delta T'} \tag{21}$$

The electric resistivity is obtained using (20) by multiplying R_m per A/L, where L and A are the length and area of the specimen under test, obtaining the equation (22). Also, the thermal conductivity of the thermoelectric material κ is obtained through (21) by multiplying K_0 per L/A, obtaining the equation (23).

$$\rho = \left[\frac{V_{max}}{I_{max}} \frac{(T_h - \Delta T_{max})}{T_h}\right] \left(\frac{A}{L}\right) \tag{22}$$

$$\kappa = \left(\frac{\alpha^2 \bar{T} \Delta T}{R_m \Delta T'}\right) \left(\frac{L}{A}\right) \tag{23}$$

Figure of Merit

As it is known, the figure of merit is calculated using (1). However, by substituting (21) into (1), a new expression is found for the figure of merit and is written utilizing the mathematical equation (24).

$$Z\bar{T} = \frac{\alpha^2 \bar{T}}{R_m K_0} = \frac{\Delta T'}{\Delta T} \quad (24)$$

Solving $\Delta T'$ from equation (9) it is found that $|\Delta T'| = |\Delta T - \Delta T_{C_C}|$ and substituting into (24) is obtained (25). Since $|\Delta T_{C_C}|$ is the result of the temperature contribution of each thermal contact, $T_{C_{hot}}$ and $T_{C_{cold}}$, where $T_{C_{cold}} = T_c$(°C) and $T_{C_{hot}} = T_h$(°C), more expressions are found as shown in (26), (27), (28), and (29).

$$Z\bar{T} = \frac{\Delta T - \Delta T_{C_C}}{\Delta T} \quad (25)$$

$$Z\bar{T}_{TEC} = \frac{\Delta T - T_c}{\Delta T} \quad (26)$$

$$Z\bar{T}_{TEG} = \frac{\Delta T - T_h}{\Delta T} \quad (27)$$

$$Z\bar{T}_{TEC} = 1 - \frac{T_c}{T_h - T_c} \quad (28)$$

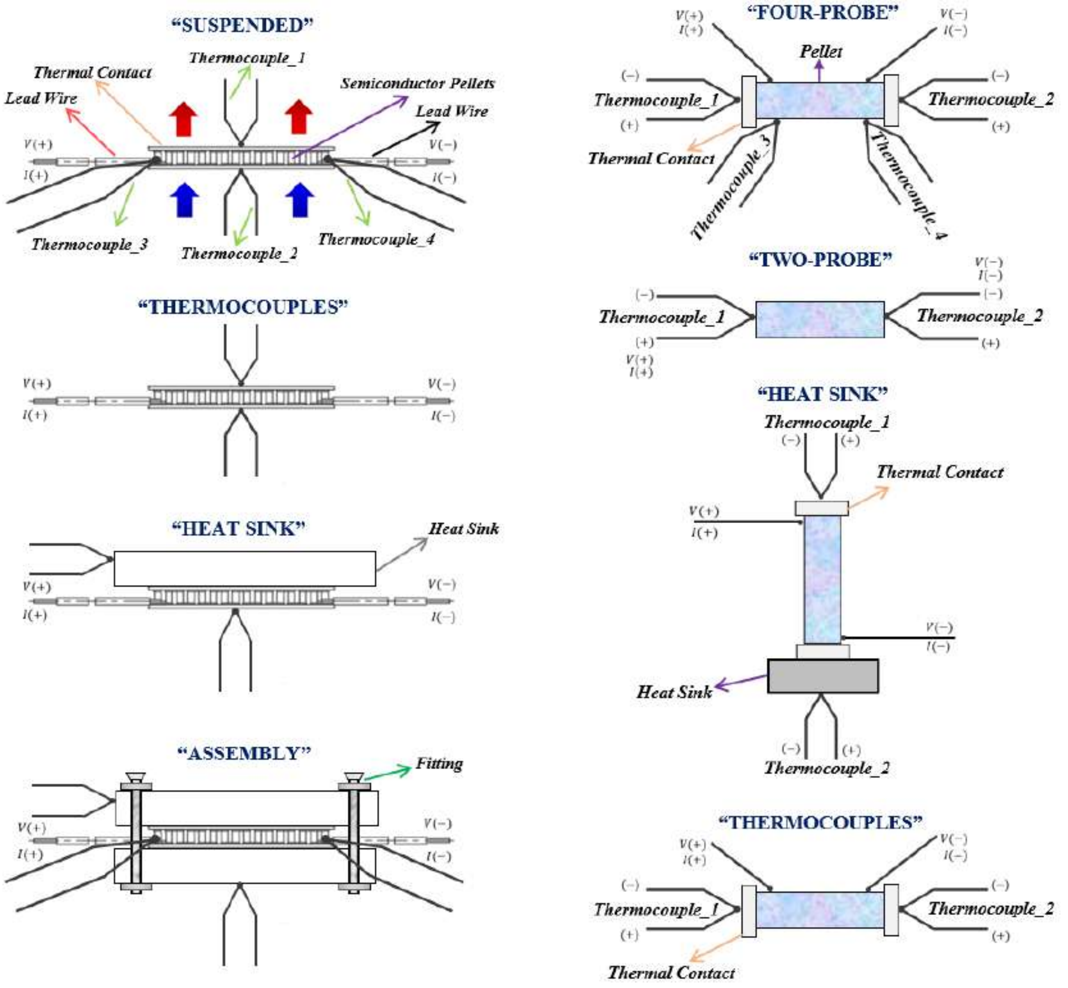
$$Z\bar{T}_{TEG} = 1 - \frac{T_h}{T_h - T_c} \quad (29)$$

The new expression for the figure of merit of TEM obtained in this research; e.g., $Z\bar{T} = \Delta T'/\Delta T$ has the form of the Harman equation $Z\bar{T} = V_\alpha/V_p$, but the $Z\bar{T}$ equation obtained herein is expressed as a function of temperatures and the one obtained by T. C. Harman is expressed as a function of voltages [15]. Equations (25), (26), and (27) for the figure of merit reciprocally have the form of the equation $Z\bar{T} = V_{oa}/(V_{ia} - V_{oa})$, obtained by R. Buist [19]. Furthermore, the second term on the right-hand side of equation (28) corresponds to the expression obtained by A. F. Ioffe [12], to measure the maximum performance ϵ_{max} of the TEM; that is, $\epsilon_{max} = T_1/(T_0 - T_1)$, where $T_1 = T_c$ and $T_0 = T_h$.

III. CHARACTERIZATION CONFIGURATIONS

A. Test Configuration for Thermoelectric Modules

Through Figure 1 (a), four different test configurations for a thermoelectric module are illustrated. The "*Suspended*" configuration employs the four probes test technique. The method used to adhere the thermocouples to the thermal contacts is simply to apply a tiny dab of thermal paste where the thermocouple junctions are positioned and can be held in place under compression with the use of thermal insulation tape [5] [19]. The "*Suspended*", "*Heat Sink*" and "Assembly" configurations are essentially the same because these configurations employ the four probes test technique. The "*Thermocouples*" configuration provides the ultimate solution in simplicity, connections, and speed of testing. This configuration is especially recommended when TEMs are used as thermogenerators. The main parameter measured using this setup is $Z\bar{T}$. This is enough to ensure the production quality of thermoelectric modules.



(a) (b)

Fig 1. Characterization configurations for thermoelectric modules (a). Characterization configurations for thermoelectric pellets (b).

This test is very effective for screening samples before more rigorous testing. Additionally, with this configuration, the characteristic thermoelectric times and relaxation times can be determined, using τ_{TE}, τ_{th}, and τ_C. Furthermore, since a ± 1.1°C error in the absolute temperature, $\bar{T}$, produces an error of less than 1%, thus reducing the uncertainty of the measurement and the figure of merit, and $Z\bar{T}$ is measurable directly with precision and versatility. The advantage of this configuration for the thermoelectric module testing is that the thermocouples will not be in contact with the active circuit and voltage pick-up will not be a factor.

B. *T*est Configuration for a Thermoelectric Pellet Sample

Fig. 1 (b), illustrates four different test configurations for a thermoelectric pellet sample. The first configuration is a suspended sample using the "*Four Probes*" technique. This test method is necessary whenever the contact resistance is unknown or significant compared to the resistance of the thermoelectric pellet. The difficulties with this configuration are that: (a) current flow through the pellet can be disturbed by the presence of the thermocouples; (b) voltage pick-up in the probes can result in significant errors in the thermocouple readings; (c) precise measurements of the probe separation are usually very difficult to obtain; and (d) the voltage and temperature planes are affected by the probes and, therefore, are not nearly as well defined at the probes as they are at the opposite ends where high-conductivity end caps are applied.

The "*Two Probes*" setup is suggested for most thermoelectric materials where good contacts are relatively easy to achieve. However, care must be taken to place the current and thermocouples on opposite edges of the end caps to avoid voltage pick-up across the thermocouples. Essentially, the thermocouple should not be placed in a position on the end cap where the current lines intersect. The "*Heat Sink*" configuration is practically the same as the "*Four-Probe*" one concerning instrumentation and connections. The "*Thermocouples* " is recommended to measure $Z\overline{T}$, τ_{th}, and the relaxation time of the material; e.g., $5\tau_{th}$.

IV. EXPERIMENTAL RESULTS

To illustrate the characterization and testing of TEMs a commercially available thermoelectric module is considered as a sample, specifically, the Kryotherm TB-127-1.4-1.2, used by Lineykin and Ben-Yaakov [3] [4]; as well as used by Y. Apertet and H. Ouerdane [11]. The parameters at $\Delta T = 70\ K$ are presented in Table 1. And the Kryotherm TB-127-1.0-1.2 module was also tested [25].

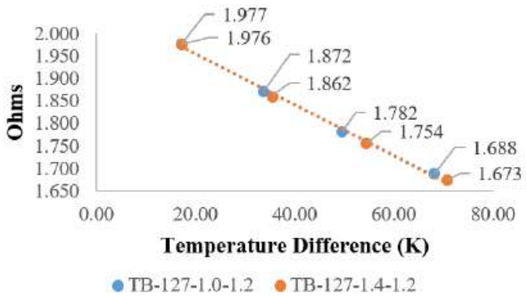
Table1. Parameters at $\Delta T = 70\ K$, $\overline{T} = 300K$, $V_{max} = 15.9\ V$, $I_{max} = 7.6\ A$, $R_{ac}\ (295K) = 1.5\ \Omega$ (Tolerance: +/- 10 %), $Q_{max} = 75\ W$.

$R_m(\Omega)$	$K_0\ (W.K^{-1})$	$C_{th}(J.K^{-1})$	$\alpha\ (V.K^{-1})$	ZT
1.602	0.667	0.35	0.0532	0.795

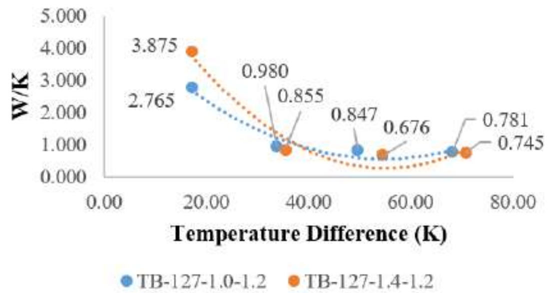
The Fig. 2 (a), (b), (c), and (d) show the test data taken on TB-127-1.0-1.2 and TB-127-1.4-1.2 using the "*Suspended*" and "*Thermocouples*" configuration without being thermally insulated; that is, in non-adiabatic conditions. The environment was at room temperature $\overline{T} = 300K$, to which the module thermal contacts were exposed. For the temperature measurements, two special type K thermocouples were used, with an error (*Special Limits Error*) of either +/- 0.25 ℃ or $+/-\ 0.4$ %. The measurement process used is similar to the one proposed by Buist in [20]. The results obtained for TB-127-1.4-1.2 at $\Delta T = 70\ K$ are shown in the Table 2, and can be compared with Table I. Additionally, analyzing the data taken on both TEM has been possible to distinguish that using the same power supply values setup (voltage, current, and time step) [25], either the TB-127-1.0-1.2 and the TB-127-1.4-1.2 showed different thermoelectric parameters. The errors are associated with the offset of thermocouples and the time step of the power source.

Table 2. Parameters at $\Delta T = 69.25\ K$, $\overline{T} = 300K$, $V_{max} = 15.9\ V$, $I_{max} = 7.6\ A$, $R_{ac}\ (295K) = 1.5\ \Omega$ (Tolerance: +/- 10 %), $Q_{max} = 75\ W$.

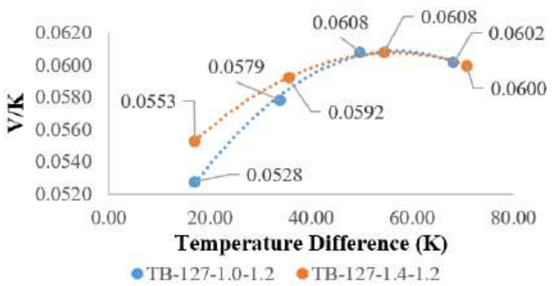
$R_{TE}(K/W)$	$C_{TE}(J/K)$	$R_m(\Omega)$	$C_{th}(J/K)$	$K_0\ (W/K)$	$\alpha\ (V/K)$	ZT
0.394	2.871	1.592	0.336	0.633	0.0508	0.768



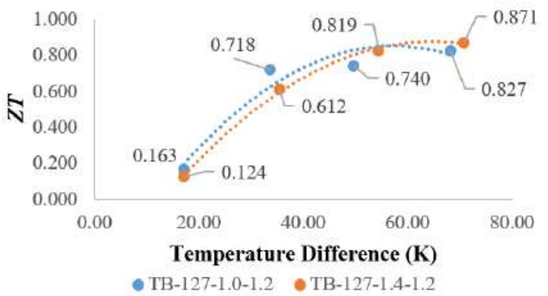
(a)



(b)



(c)



(d)

Fig. 2. Electric resistance characterization data taken on TB-127-1.0-1.2 and TB-127-1.4-1.2 (a). Thermoelectric conductance characterization data taken on TB-127-1.0-1.2 and TB-127-1.4-1.2 (b). Seebeck coefficient characterization data taken on TB-127-1.0-1.2 and TB-127-1.4-1.2 (c). Figure of merit characterization data taken on TB-127-1.0-1.2 and TB-127-1.4-1.2 (d).

CONCLUSIONS

The unification of the forced response and natural response theories allows us to describe the complete response of TEMs. From the complete response of TEMs, it has been possible to create the basis to develop and describe in detail a novel methodology for the characterization and testing of thermoelectric materials, pellets, wafers, ingots, modules, and systems. The present method is proven to be fast, accurate, highly appropriate to apply, and conveniently more cost-effective. Using an integrated measurement system that is capable of solving simultaneous the voltage and temperature components of TEMs with high speed and high resolution. Moreover, this methodology provides all the advantageous features of Harman's and Transient's methods. Numerous expressions for the figure of merit expressed as a function of the temperatures were found similar to Harman's and Buist's equations. The methodology provided a direct measurement of the figure of merit with exactness and reproducibility. The subsequent computations yield values for the Seebeck coefficient, electrical resistance, thermal conductance, as well as the famous thermoelectric transport coefficients ρ and κ.

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