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Mathematical model to evaluate the stresses and strains of oil strings in transient state

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Abstract. - This study presents the development of a mathematical model to assess the performance of oil strings during the drilling process of a reservoir, considering the dynamic conditions and operational characteristics of the equipment during its functions, and taking into account the mechanical properties of API K55 steel. The research resulted in a set of equations that model the behavior of stresses and deformations experienced by the oil tools when transmitting torque, facilitating the opening of the reservoir. A finite element analysis was conducted to evaluate the structural behavior of the strings and to estimate the time required to reach permanent deformations, as well as the time before failure occurs.

Keywords: oil string, deformation, stresses, lifespan.

Modelo matemático para evaluar las tensiones y deformaciones de las sartas petroleras en estado transitorio

Resumen: Este estudio presenta el desarrollo de un modelo matemático para evaluar el desempeño de las sartas petroleras durante el proceso de perforación de un reservorio, considerando las condiciones dinámicas y características operacionales del equipo durante sus funciones, y teniendo en cuenta las propiedades mecánicas del acero API K55. La investigación dio como resultado un conjunto de ecuaciones que modelan el comportamiento de las tensiones y deformaciones experimentadas por las herramientas petrolíferas al transmitir el par de torsión, facilitando la apertura del yacimiento. Se realizó un análisis de elementos finitos para evaluar el comportamiento estructural de las sartas y estimar el tiempo necesario para alcanzar deformaciones permanentes, así como el tiempo antes de que se produzca el fallo.

Palabras clave: sarta petrolera, deformación, tensiones, vida útil.

I. INTRODUCTION

A. *Formulation of the Problem*

The global development of petroleum exploration necessitates the utilization of drilling equipment capable of accessing oil reserves in the Earth's deepest strata. This process requires high-technology apparatus that delivers performance commensurate with the operational demands of the work environment, where equipment is exposed to diverse climatic conditions [1].

Contemporary drilling tools are engineered to function in adverse weather conditions and varied geological formations. A notable advancement in this field is deep-water drilling technology, which has progressed significantly, enabling exploration at depths exceeding 300 meters. This progress is attributable to innovations in tools such as pressure nozzles and rotary steerable systems (RSS). These technological advancements facilitate more efficient and secure operations, thereby reducing operational time and associated costs [2], [3].

To address the issue of deformation failures in oil strings, it is crucial to comprehend the mechanical and operational challenges these tools encounter during reservoir opening operations. Oil strings are subjected to extreme load conditions due to continuous interaction with rock formations and drilling fluids. Recent studies have emphasized the importance of optimizing string design to enhance resilience and mitigate the likelihood of premature failure [4].

Research in drilling engineering has revealed that the dynamic behavior of strings is critical to their longevity [4]. Mathematical models and simulations have demonstrated that torsional and axial vibrations can accelerate wear on the walls of petroleum tools and material fatigue. Furthermore, the implementation of advanced real-time monitoring techniques enables the detection of anomalies in the machine element's behavior, facilitating preventive interventions that can extend its service life and improve drilling operation efficiency.

Moreover, it has been observed that strings begin to experience wear and deformation over time once they have completed their useful life cycle in drilling processes [5]. This creates a latent problem during operational activities, with potential failures compromising the progress of reservoir opening. In response to this issue, a transient state mathematical model is being developed to analyze the behavior of stresses and deformations produced by the drive system that provides axial advancement of the oil string during its functional performance.

B. *Literature Review*

Aquí está la traducción en tono académico de los párrafos proporcionados:

Certain authors [5] analyze pressure gradients during string advancement operations, presenting models to predict sequences and magnitudes of these pressure peaks. They note that the greatest uncertainties in the models are related to viscous friction losses, which lead to estimated parameters associated with the calculation of friction forces. It is emphasized that, due to limitations in real-time pressure measurement during reservoir opening operations, the model utilizes bottom-hole pressures. This approach allows for model calibration in consecutive operations if pumps are activated prior to opening procedures.

Other investigations highlight that vibrations can be categorized into three distinct behaviors: whirl (lateral), stick-slip (torsional), and bit bounce (axial) [6]. These phenomena can cause tool failures, increasing operational costs due to drilling time delays. They also emphasize that optimization is not solely about obtaining parameters that provide the highest rate of penetration (ROP), but rather seeking parameters that allow for faster and more efficient drilling. The drilling speed test curve is utilized to obtain parameters that aid in process optimization, not only maximizing ROP but seeking the optimal combination of highest penetration rate and shortest operational time under safe conditions for both equipment and operating personnel.

Some mathematical models allow for the prediction of stress sequences and magnitudes, both positive and negative, due to hydrocarbon movement in strings [7]. These models are essential for understanding how these phenomena occur and mitigating their effects on oil transfer operations. Furthermore, these studies [7] consider the seismic response of fractured reservoirs, which is crucial for identifying and characterizing fractures in deep carbonate formations. This information is valuable for optimizing hydrocarbon extraction and minimizing operational risks.

Moreover, reference [8] mentions an analysis of the seismic characteristics of hydrocarbon reservoirs in the Tarim Basin, particularly the anomalous response known as "string of bead-like response" (SBLR). It is shown that precise identification of these reservoirs is crucial for efficient drilling, considering its impact on string design and opening operation parameters. Understanding the location and geological formation, along with reservoir quality, aids in efficient drilling planning and appropriate string selection. Additionally, the physical models developed in [8] provide valuable information for the design and optimization of oil strings, especially in complex and fractured environments where drilling risks are higher.

Other research [9] focuses on analyzing the mechanical behavior of strings due to the complexity generated by the combination of temperature and pressure in reservoirs, as well as loads induced by steam injection and production operations. Furthermore, an analytical model based on energy conservation and heat transfer principles is established. The study calculates temperature and pressure fields during these operations. Force distribution analysis and strength verification of the integrated injection and production string were performed. Additionally, fatigue damage was evaluated considering dynamic loads during the injection process, and corrosion life was predicted, showing these as the main causes of string failures. The significant reduction in residual strength due to corrosion was identified as the fundamental factor in failures.

II. THE PHYSICAL MODEL OF OIL DRILL STRINGS

Oil strings are cylindrical hollow drilling tools, as shown in Figure 1, whose main role is to enable the axial advancement of the drill bit during the opening of a reservoir. These elements, while performing their functions, experience the application of a high driving torque originating from the rotary table located in the drilling rig.

Figure 1. General model of the oil well string [10].

For the development of the mathematical model, the finite element differential analysis was applied [11], [12], based on the following considerations to be taken into account in the field of applications of the equations to be deduced:

- The oil string is made of ductile material.
- The steel of the drill string has a homogeneous composition.
- The general normal stresses are within the elastic range.
- A uniform cross-section along its length.
- The stresses generated during torque application do not exceed the yield strength of the material.
- A linear distribution of shear stresses is assumed due to the transverse symmetry of the string.
- It is considered that the fixed end experiences no angular or shear deformation.
- A uniform torque distribution along the string is assumed.

A hollow circular shaft is connected to a fixed support represented by the brown circle at one of its ends, as seen in Figure 2, and a torsional moment T is applied to the other end. The shaft will deform as its free end rotates, generating a rotation angle called θ, due to the angular displacement experienced by point O located at the free end as a result of the torque action. Similarly, the shaft experiences longitudinal shear deformations denoted as γ. This example, presented in Figure 2, occurs when the drill string becomes stuck during its functions in the opening of a reservoir, and the transmission remains engaged, transmitting a torsional moment to the string.

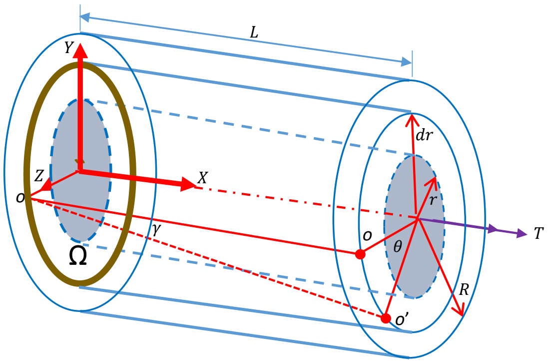

Figure 2. Differential element of the oil string.

III. METHODOLOGY

A. Initial and boundary conditions:

The Dirichlet condition states that all infinitesimal elements located at the left end of the bar shown in Figure 2, will remain with all their degrees of freedom constrained and equalized zero during the analysis time, i.e. for t=0. It means, the translation and rotation displacements will be zero in any direction for all points located in the domain Ω belonging to the YZ plane defined by the circumferences of the outer and inner edge of the string.

- ✓ Translation offset in x-direction $u = 0 \ and \ \theta_x = 0$ (1)
- ✓ Translation offset in y-direction: $v = 0 \ and \ \theta_y = 0$ (2)
- ✓ Translation offset in z-direction: $w = 0 \ and \ \theta_z = 0$ (3)

The Newman condition considers that the derivatives of the displacements in the domain Ω of the end of the string that remains embedded, will be zero because those points have all their degrees of freedom are constrained therefore it is expressed as follows:

In x- axis direction: $\frac{\partial u}{\partial x} = 0 \ y \ \frac{\partial u}{\partial t} = 0$ (4)

In y- axis direction: $\frac{\partial v}{\partial x} = 0 \ y \ \frac{\partial v}{\partial t} = 0$ (5)

In z-axis direction: $\frac{\partial w}{\partial x} = 0 \ y \ \frac{\partial w}{\partial t} = 0$ (6)

The analysis of the displacement experienced by the point o towards o' analytically is obtained from the following equation, starting from figure 2:

$$\gamma . L = oo' = \theta_x . r \quad (7)$$

Where γ is the shear deformation, L is the axial length of the bar, r is the inner radius of the cylinder. Therefore, the shear deformation (γ) is given by:

$$\gamma = \frac{\theta_x . r}{L} \quad (8)$$

The evaluation of the shear deformation at the outer radius yields the following results

$$\gamma_{max} = \frac{\theta_x . R}{L} \quad (9)$$

Equating the angular deformation θ present in equations (8) and (9)

$$\gamma = \frac{\gamma_{max} . r}{R} \quad (10)$$

Where $\gamma_{max}$is the maximum shear deformation.

Under the considerations present in the development of the model, we proceed to apply Hooke's law following equation (11)

$$G . \gamma . = G \frac{\gamma_{max} . r}{R} \quad (11)$$

Where G is the modulus of rigidity of the material, which allows defining the distribution of shear forces present in the tubular section.

$$\tau = \tau_{max} . \frac{r}{R} \quad (12)$$

Where τ: Shear stress, τ_{max}: Maximum shear stress, r : Cylinder inner radius, R: Cylinder outer radius. We will consider the infinitesimal element in the shaft cross section when a uniform torque is applied to it as shown in fig. 3.

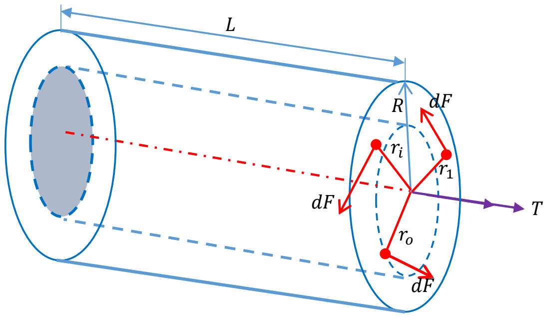

Figure. 3. Infinitesimal analysis of a driving shaft.

Therefore, the torsional moment T is defined:

$$T = \int_0^R r.dF \qquad (13)$$

Donde T: torsional moment, dF: Differential force, r: Cylinder inner radius, R: Cylinder outer radius. Bearing in mind that the shear stress [13] is defined by (14):

$$\tau = \frac{dF}{dA} \qquad (14)$$

Where: τ: Shear stress, dA: Area differential, dF: Force differential.

Replacing (14) in (13) we obtain:

$$T = \int_0^R r.\tau.dA \qquad (15)$$

Substituting (12) in (15) results in:

$$T = \int_0^R r.\tau_{max}.\frac{r}{R}.dA \qquad (16)$$

Bearing in mind that the outer radius is constant and the maximum shear stress remains constant at the outer edge of the string, it is obtained:

$$\mathrm{T}\, T = \frac{\tau_{max}}{R}\int_0^R r^2.dA \qquad (17)$$

Considering that the area of a cylinder is defined by $A = \pi.r^2$, therefore $dA = 2\pi.r.dr$ substituting (17)

$$T = \frac{\tau_{max}}{R}\int_0^R 2\pi.r^3.dr \qquad (18)$$

Solving the integral yields $J = \frac{\pi}{2}R^4$, being this the polar moment of inertia, in this sense the maximum shear stress is defined by:

$$\tau_{max} = \frac{T.R}{J} \qquad (19)$$

Where τ_{max}: Maximum shear stress, R: Cylinder outer radius, J: Polar moment of inertia.

IV. RESULTS

By considering that the drive of the oil string, during the opening of a reservoir requires the application of a power coming from the driving element, called rotary table, and that the string represents a tubular shaft of circular section, used to transmit the power to the oil drill bit, as shown in Figure 4. When used for this purpose, it is subjected to a torsional moment that depends on the power generated by the machine and the angular velocity of the shaft.

Figure 4. Oil drillstring assembly with drill bit.

Power Pot: is defined as the work done per unit of time. In turn, the work transmitted by a rotating shaft is equal to the torque applied times the angle of rotation, Therefore, if during an instant of time dt a torque T a is applied causing the string to rotate an angle $d\theta_x$, then the instantaneous power is:

$$Pot = T.\frac{d\theta_x}{dt} \qquad (20)$$

Since the angular velocity of the axis $w = \frac{d\theta_x}{dt}$ the power can be expressed as follows:

$$Pot = T.w \qquad (21)$$

Where
T: Torsional moment (torque); w: Angular velocity of the bar; Substituting the torque of eq. 21 in eq. 10, we obtain:

$$\theta_x = \frac{Pot.L.dt}{J.G.d\theta_x} \qquad (22)$$

Integrating with respect to θ_x we obtain:

$$\theta_x = \left(\frac{2.Pot.L}{J.G}.t\right)^{1/2} \qquad (23)$$

which represents angular deformation of the oil string in transient state.

In a similar way, the shear deformation in transient state is determined by substituting eq.24 in eq.9 and rearranging to obtain:

$$\gamma_{max} = \left(\frac{2.Pot.R^2}{J.G.l}.t\right)^{1/2} \qquad (24)$$

To determine the equation of shear forces in transient state, Hooke's law is applied, multiplying on both sides of eq.25 by the stiffness modulus and simplifying the equation to the following form:

$$\tau_{max} = \left(\frac{2.Pot.G.R^2}{J.L}.t\right)^{1/2} \quad (25)$$

Considering that polar moment of inertia for hollow cylinders is defined by:

$$J = \frac{\pi}{2}(R^4 - r^4) \quad (26)$$

Substituting the equation of the polar moment of inertia defined by eq.27 in eq.26 and simplifying it remains:

$$\tau_{max} = \left(\frac{4.Pot.G.R^2}{\pi.(R^4 - r^4).L}.t\right)^{1/2} \quad (27)$$

Graphical representation of the stress state, generated at the outer edge of the oil string as can be visualized in Figure 5, being α the principal plane to be determined, once Mohr's circle is applied.

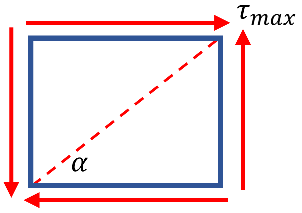

Figure 5. Stress state of an infinitesimal element located at the outer edge of the string.

Bearing in mind that Mohr's circle allows to obtain the normal and maximum shear forces, as well as the principal plane, where the normal stress is maximized, we proceed to define the coordinates of the circle in a generic way, considering that the coordinates of point 1 is given by : Pto1 = $\sigma_x, -\tau_{xy}$), whereas point 2 is expressed by the relation Pto 2 = σ_y, τ_{xy}) these two points form a diagonal that passes through the center of the circle, therefore the center is given by $C = \frac{\sigma_x+\sigma_y}{2}$, therefore the radius of is defined by:

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} \quad (28)$$

Applying the theory related to Mohr's circle to the state of stress shown in Figure 5, the following is obtained:

- Coordinate of point 1 = $0, -\tau_{xy}$), since there are no normal forces in x direction.
- Coordinate of point 2 = $(0, \tau_{xy})$, since there are no normal stresses in the y direction.
- Center coordinate C = (0,0) since there are no normal forces in x or y direction.
- Radius magnitude: $R = \sqrt{\left(\frac{0-0}{2}\right)^2 + \left(\tau_{xy}\right)^2} = |\tau_{xy}|$

The graphical representation of Mohr's circle (Figure 6) is made, in order to know the maximum values of normal and shear forces as shown in Figure 6, I feel $\tau_{xy} = \tau_{max}$ defined by (28).

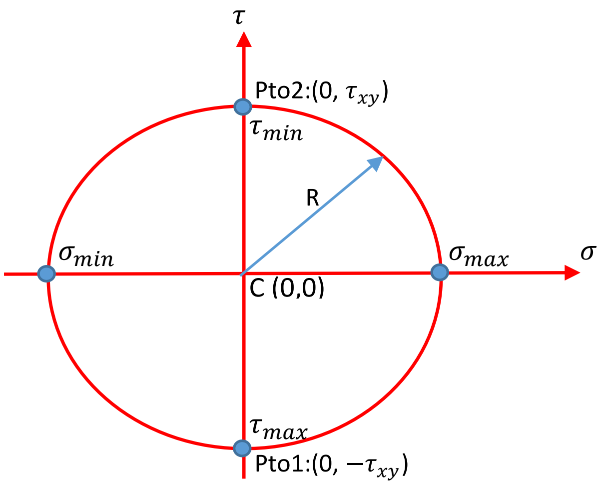

Figure 6. Mohr's circle.

Based on the results obtained through Mohr's circle, we conclude the following:

The maximum normal and shear stress generated in the oil string in a transient state, when a torque is supplied, is defined by the following equation:

$$\sigma_{max} = \tau_{max} = \left(\frac{4.Pot.G.R^2}{\pi.(R^4 - r^4).L}.t\right)^{1/2} \tag{29}$$

The proposed mathematical model allows to determine the useful life time of an oil string known the operating power during the opening of a reservoir, considering the yield stress of the tool steel, also allows continuous monitoring of how the maximum stress defined by equation (29) increases over time, once this exceeds the yield stress of the steel it experiences permanent deformations that can lead to premature failure of the string in the process of operation.

A. Analysis of an oil string with an API K55 steel

Table 1 shows the mechanical properties of API K55 steel, including stiffness, yield stress and ultimate stress.

Table 1. Mechanical properties of API K55 steel.

Steel type	API K55	Unids
Stiffness modulus	205	Gpa
Yield stress	552	Mpa
Ultimate stress	655	Mpa

For the purpose of this work, we have an oil string with the following technical operating information (Table 2).

Tabla 2. Dimensions of the string.

Parameter	Dimension	Unid
Outer radius	0.12	m
Inner radius	0.10	m
Drilling depth	2000	m
Drive power	20000	watt

Considering the dimensional parameters and operations of the oil string, we proceeded to evaluate the mathematical model of the stresses in transient state defined by equation (30), for a time interval from 0 to 2500 seconds, being this time enough for the element to fail due to shear stresses, the results obtained are shown in Table 3 below.

Table 3. Transient state stress analysis.

Time (Seg)	Normal stress. [Pascal.]	Factor of safety.	Time (Sec)	Normal stress [Pascal.]	Factor of safety-
0	0.00E+00	0	1300	6.75E+08	9.71E-01
100	1.87E+08	3.50E+00	1400	7.00E+08	9.36E-01
200	2.65E+08	2.48E+00	1500	7.25E+08	9.04E-01
300	3.24E+08	2.02E+00	1600	7.48E+08	8.75E-01
400	3.74E+08	1.75E+00	1700	7.71E+08	8.49E-01
500	4.18E+08	1.57E+00	1800	7.94E+08	8.25E-01
600	4.58E+08	1.43E+00	1900	8.16E+08	8.03E-01
700	4.95E+08	1.32E+00	2000	8.37E+08	7.83E-01
800	5.29E+08	1.24E+00	2100	8.57E+08	7.64E-01
900	5.61E+08	1.17E+00	2200	8.78E+08	7.46E-01
1000	5.92E+08	1.11E+00	2300	8.97E+08	7.30E-01
1100	6.21E+08	1.06E+00	2400	9.17E+08	7.15E-01
1200	6.48E+08	1.01E+00	2500	9.36E+08	7.00E-01

Figure 7 shows the distribution of shear stress as a function of time, highlighting an increase in the magnitude of stresses over time. This type of analysis allows for the estimation of the service life of the element under dynamic operating conditions during the opening of an oil reservoir, where the variation of normal stresses occurs as the operation type progresses with the drill bit stuck in the formation. In other words, the structural element of the drill string experiences normal stresses in short time intervals that may lead to the fracture of the tool under dynamic conditions.

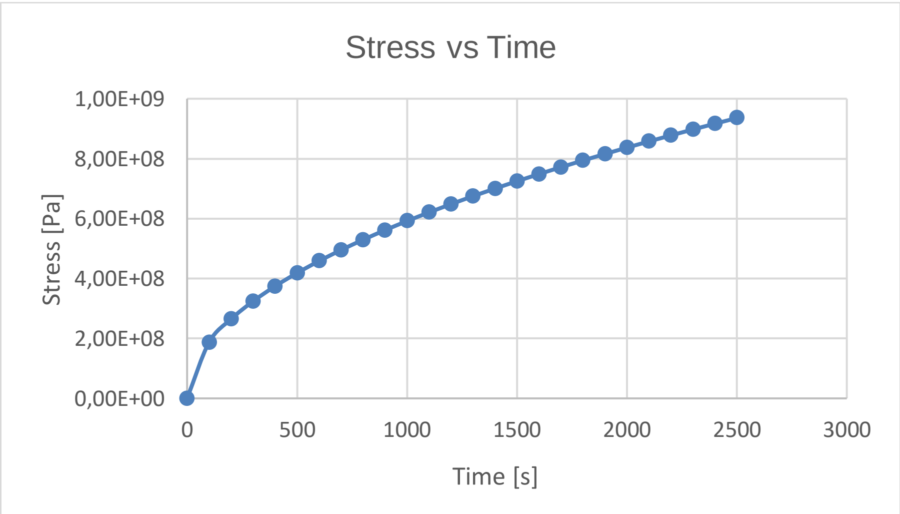

Figure 7. Shear stress distribution as a function of time.

B. Estimation of time of deformation and failure of the oil string

Considering that the steel of the API K55 type oil strings, with mechanical properties that have a yield stress of 552 Mpa and an ultimate stress of 655 Mpa as shown in Table 1, where these do not vary over time, a graphical relationship can be obtained that allows estimating the time it would take for the string to overcome the elastic deformation, as well as the time required to generate a shear failure as shown in Figure 8.

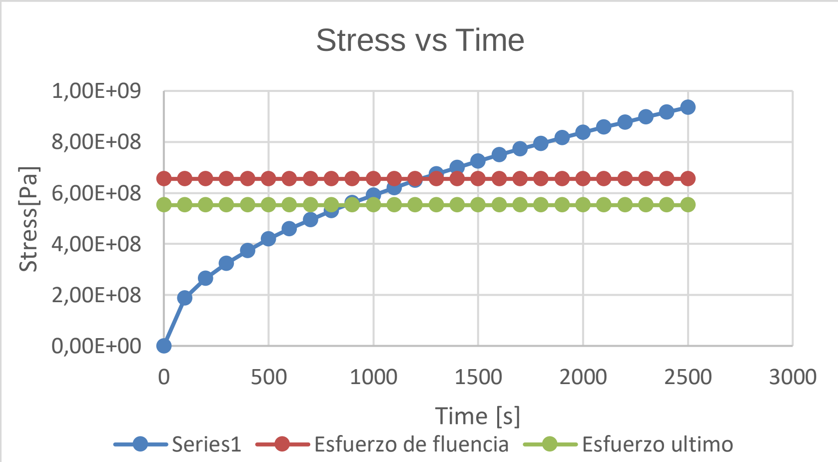

Figure 8. Estimated useful life.

It is evident that the permanent deformations begin to appear after a time lapse of 900 seconds, similarly it is visualized in Figure 8 that the time required for the string to break due to excess of shear stresses generated in the structure is 1200 seconds, that is to say that in case the bit becomes stuck during drilling, the operator has a time of less than 20 minutes to release the bit from the grip and keeping the rig transmission on, exceeding this time the oil string will tend to break due to the maximum capacity of the normal stresses offered by the API K55 steel.

C. Safety Factor Analysis

In any drilling process, the safety of the equipment used during the opening of oil reservoirs must prevail. For this reason, Figure 9 presents the safety factor as a function of the operational time of the drill string under dynamic conditions. The safety factor is considered an indicator of the ultimate stress ratio to the stress generated by dynamic conditions. It is important to highlight that this indicator should be greater than one to ensure the operability of the string. If it is less than one, it indicates failure due to exceeding the maximum capacity that the steel can provide, which has been surpassed by the stresses produced by the effects of the operational parameters.

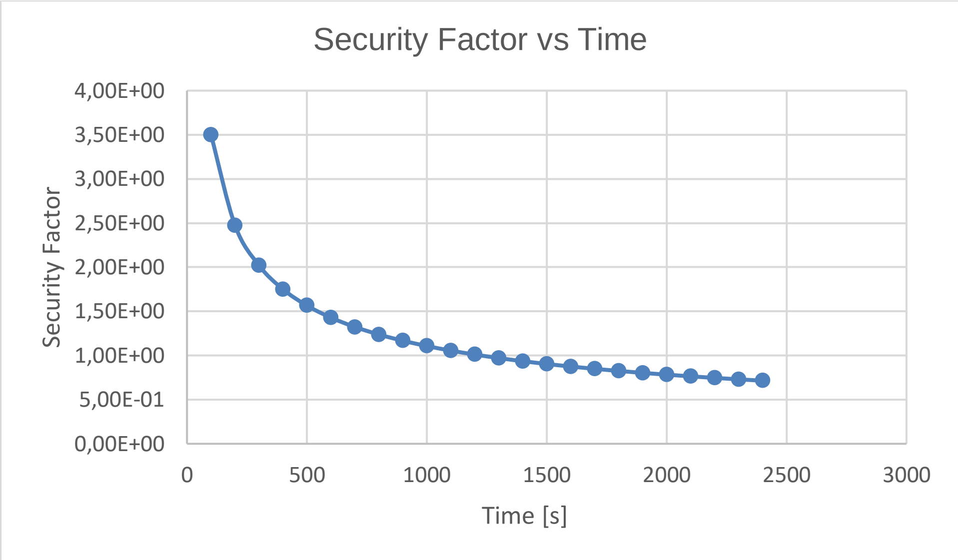

Figure 9. Factor of safety vs. time.

Figure 9 shows that the material of the drill string reaches its maximum resistance capacity after 1200 seconds. After this time, the stresses exceed the capacity of API K55 steel, and the element will experience failure due to rupture. This operational graph provides the operator with information on the safe maneuvering time for the drill in the event of the drill string becoming stuck during the opening of a reservoir, thereby saving time by preventing a failure that could prolong the work of opening a hydrocarbon reserve.

CONCLUSIONS

The principal plane where maximum normal stress occurs is located at 45° with respect to the horizontal plane. This indicates that if the drill string were to fail due to excessive normal stresses, the crack would tend to form at an angle corresponding to the principal plane. This phenomenon is critical for the structural integrity of the string, especially under conditions of high pressure and cyclic loading.

A mathematical model has been established to predict the deformations and stresses generated in a transient state in drill strings when a driving torque is applied during the opening of an oil reservoir. This model integrates differential equations that describe the dynamic behavior of the string under the influence of external forces and specific boundary conditions.

This mathematical model represents a significant advancement in the ability to predict and manage deformations and stresses in drill strings during the transient state of reservoir opening, contributing to structural and operational integrity in the oil industry.

The time required for drill strings to reach a permanent angular deformation for API K55 steel was determined to be 900 seconds. This value was obtained through experimental analysis involving the application of constant torsion to steel samples under controlled conditions.

The results obtained are critical for the planning and operation of drill strings, as permanent deformation can affect the structural integrity and operational efficiency of the string. The time of 900 seconds serves as a reference parameter for drilling engineers, allowing them to design procedures and select materials that minimize the risk of permanent deformation during drilling and production operations.

The service life of the drill string under the previously defined dynamic conditions in our analysis was established at 1200 seconds for steel that meets the API K55 standard in the event of a stuck drill bit with the transmission engaged. This value was determined using a combined approach of experimentation and numerical modeling, evaluating the behavior of the string under conditions of cyclic loading and extreme torsional stresses. The study also suggests that optimizing the design of the string, such as selecting materials with greater fatigue resistance and implementing protective coatings against corrosion, can extend the service life beyond 1200 seconds under adverse operating conditions.

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